自然単位系

$$c = \hbar = 1$$

[長さ] = [時間]

[質量] = [運動量] = [エネルギー]

[エネルギー] = [時間]$^{-1}$

$$\psi(t) \sim e^{-iEt + ip\cdot x}$$

換算

$$c = 3 \times 10^{10}\ \mathrm{cm\ s^{-1}}$$

$$\hbar = 6.6 \times 10^{-25}\ \mathrm{GeV\ s}$$

$$\hbar c = 0.2\ \mathrm{GeV\ fm}$$

$$(\hbar c)^2 = 0.39\ \mathrm{GeV^2\ mb}$$

$$1\ \mathrm{fm} = 10^{-15}\ \mathrm{m} = 10^{-13}\ \mathrm{cm}$$

特殊相対論

4-vector $(a^0, a^1, a^2, a^3) = (a^0, \mathbf{a})$

内積 $a\cdot b = a^0 b^0 - \mathbf{a}\cdot\mathbf{b}$ Lorentz 不変

粒子の momentum

$$(E, \mathbf{p}) = p^\mu$$

$p^2 = E^2 - \mathbf{p}^2 = m^2$: 質量

$\beta = \frac{|\mathbf{P}|}{E} = \frac{v}{c}$: 速度 $0 \le \beta < 1$

$\gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{E}{m}$ Lorentz factor $1 \le \gamma < \infty$

エネルギー $E = \gamma m$

運動量 $\mathbf{p} = \beta E = \gamma\beta m$

粒子の崩壊

τ : lifetime $n(t) = n_0 e^{-t/\tau}$ (in rest frame)

走っている粒子は寿命がのびる.

みかけの寿命 $= \gamma\tau$

走る距離 $= \gamma\beta c\tau$

(例) $p = 1\,\mathrm{GeV}$ の K^+, K_S^0 がどれだけとぶか

$m_K \sim 0.5\,\mathrm{GeV} \rightarrow \gamma\beta = \frac{p}{m} \sim 2$

$\tau_{K_S} = 0.9\times10^{-10}\,\mathrm{s} \rightarrow \gamma\beta c\tau \simeq 5\,\mathrm{cm}$

$\tau_{K^+} = 1.2\times10^{-8}\,\mathrm{s} \rightarrow$ 〃 $7\,\mathrm{m}$

t/τ	$e^{-t/\tau}$
ϵ	$1-\epsilon$
10^{-3}	0.9990
10^{-2}	0.990
10^{-1}	0.905
1	0.368
10	4×10^{-5}
100	4×10^{-44}

Why Collider ?

Beam - Fixed target

$p_1 = (E, 0, 0, E)$ $\quad$ $p_2 = (M, 0, 0, 0)$ $\quad$ $E \gg M$

$$S = (p_1 + p_2)^2 = (E+M)^2 - E^2 \cong 2ME$$

(重心系エネルギー^2)

Beam - Beam

$p_1 = (E, 0, 0, E)$ $\quad$ $p_2 = (E, 0, 0, -E)$

$$S = (p_1 + p_2)^2 = 4E^2$$

重心系 = 実験室系

Asymmetric collider

(例: HERA)

e 30 GeV

p 820 GeV

$\sqrt{S} = 314$ GeV

$(E_1, 0, 0, E_1)$ $\quad$ $(E_2, 0, 0, -E_2)$

$$S = 4E_1E_2$$

Colliderの弱点: Small rate

beam $\sim 10^{12}$ particles

solid target $\sim 10^{24}$ particles / cm^3

Colliderの実験の基本思想

あらゆるeventをできるだけ記録する

→ さまざまな解析を同じデータに基いて行う.

---

e^+e^-ではevent rateは低い

(例) $\sqrt{s} = 50$ GeV　$\sigma(e^+e^- \to hadron) \sim 0.2\,nb = 2\times10^{-34}\,cm^2$

$\mathcal{L} = 10^{31}\,cm^{-2}s^{-1}$ → event rate $= 2\times10^{-3}\,s^{-1}$ $\sim$ 10分に1回

(10^8 beam crossingに1回)

$\underbrace{\text{ほとんどは}}_{\text{のtrigger}}$ ゴミ (宇宙線muon, beam-gas events)

Special purpose detectorsも少数ながらある

ASP (PEP) : Single photon search

SHIP (TRISTAN) 他: Monopole search

加速器のエネルギー

陽子 Synchrotron

最高エネルギーは最大磁場できまる

$R = \frac{P_T}{eH}$

$\omega = \frac{eH}{E}$

$$\text{bending radius} \propto \frac{\text{momentum}}{\text{磁場}}$$

E ↑ → radius ↑ → 大きな ring

→ mag. field ↑ → 超伝導 magnet

電子 synchrotron

最高エネルギーは Synchrotron radiation できまる

$I = \frac{2e^4 H^2 p^2}{3m^4}$

$$\text{energy loss} \propto \frac{E^4}{R^2}$$

E ↑ → 電力をつぎこむ

→ radius ↑

Storage ring のコスト $\propto \underset{\substack{\uparrow \\ \text{建設費}}}{R} + \underset{\substack{\uparrow \\ \text{To keep luminosity}}}{R \cdot \frac{E^4}{R^2}} \sim \underset{\text{optimum}}{E^2}$ $(R \sim E^2)$

→ Linear collider cost ~ E

Luminosity

実験の sensitivity のめやす

$$\mathcal{L} \sim \frac{N_{e^+} N_{e^-} f\, n_B}{A}$$

Luminosity ↑

$N_{e^\pm}$: Number of $e^\pm$ per bunch ↑ カレントをふやす

f : Frequency (= c / 一周の長さ)

n_B : バンチ数 ↑

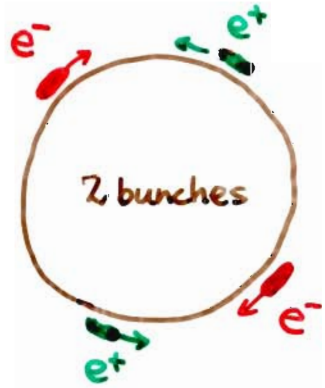
A : ビームの断面積 ↓ ビームをしぼる

単位 :

$\mathrm{cm}^{-2}\ \mathrm{sec}^{-1}$

Typical collider has

$\sim 10^{30}\ \mathrm{cm}^{-2}\mathrm{sec}^{-1}$



Luminosity × cross section = 一秒あたりの反応数

Integrated Luminosity

$\int \mathcal{L}\, dt$ Luminosity の時間積分

Integrated luminosity × cross section = 全反応数

単位

$\mathrm{pb}^{-1} = 10^{36}\ \mathrm{cm}^{-2}$

$\mathrm{nb}^{-1} = 10^{33}\ \mathrm{cm}^{-2}$

$\mathcal{L} \sim 10^{31}\ \mathrm{cm}^{-2}\mathrm{s}^{-1}$ で

1年間 ($\sim 10^7$ s)

$\rightarrow 10^{38}\ \mathrm{cm}^{-2} = 100\ \mathrm{pb}^{-1}$

Cross section

断面積 ： 反応の起こりやすさ　　　次元 ： [長さ]^2 or [エネルギー]$^{-2}$

単位　cm^2　　　$1\,\mathrm{b} = 10^{-24}\,\mathrm{cm}^2$

1 mb	$= 10^{-27}\,\mathrm{cm}^2$	← hh ← γh
1 μb	10^{-30}	
1 nb	10^{-33}	← TRISTAN
1 pb	10^{-36}	← TeV e^+e^-
1 fb	10^{-39}	

$\sigma_{pp,\,tot} \sim 40\ \mathrm{mb}$ (at minimum)

$$\sigma(e^+e^- \to \mu^+\mu^-) = \frac{4\pi\alpha^2}{3s} = \frac{87\ \mathrm{nb}}{s\,(\mathrm{GeV}^2)}$$: unit of R

(QED lowest order)

$\sqrt{s}$ = c.m. energy

→ (時間)

e^+e^- Colliding Facilities

Name	Location	Typical Energy	Major Detectors	Started
SPEAR	SLAC (Stanford)	J/ψ – 7.4	Mark I, Mark II, DELCO, Crystal Ball•, Mark III	1973
DORIS	DESY (Hamburg)	J/ψ – 7.	PLUTO, DASP, LENA•	1974
DORIS II		ϒ region	ARGUS, Crystal Ball	1978
VEPP-4	ITEP (Novosibirsk)	ϒ region		
CESR	Cornell (Ithaca)	ϒ region	CLEO, CUSB•	1980
PEP	SLAC	29	Mark II, MAC, HRS, DELCO, TPC-2γ, ASP	1980
PETRA	DESY	34. – 47	PLUTO, Mark J, TASSO, JADE, CELLO	1978 - 86
TRISTAN	KEK (Tsukuba)	50 – 57 ↑	VENUS, TOPAZ, AMY	1986
SLC	SLAC	Z	Mark II, SLD	1988 ?
LEP	CERN (Genève)	Z Phase II: 190	ALEPH, DELPHI, L3, OPAL	1989
CLIC		TeV		
JLC				
BEPC	北京	ψ region		
"B factory"	Several proposals	ϒ region		

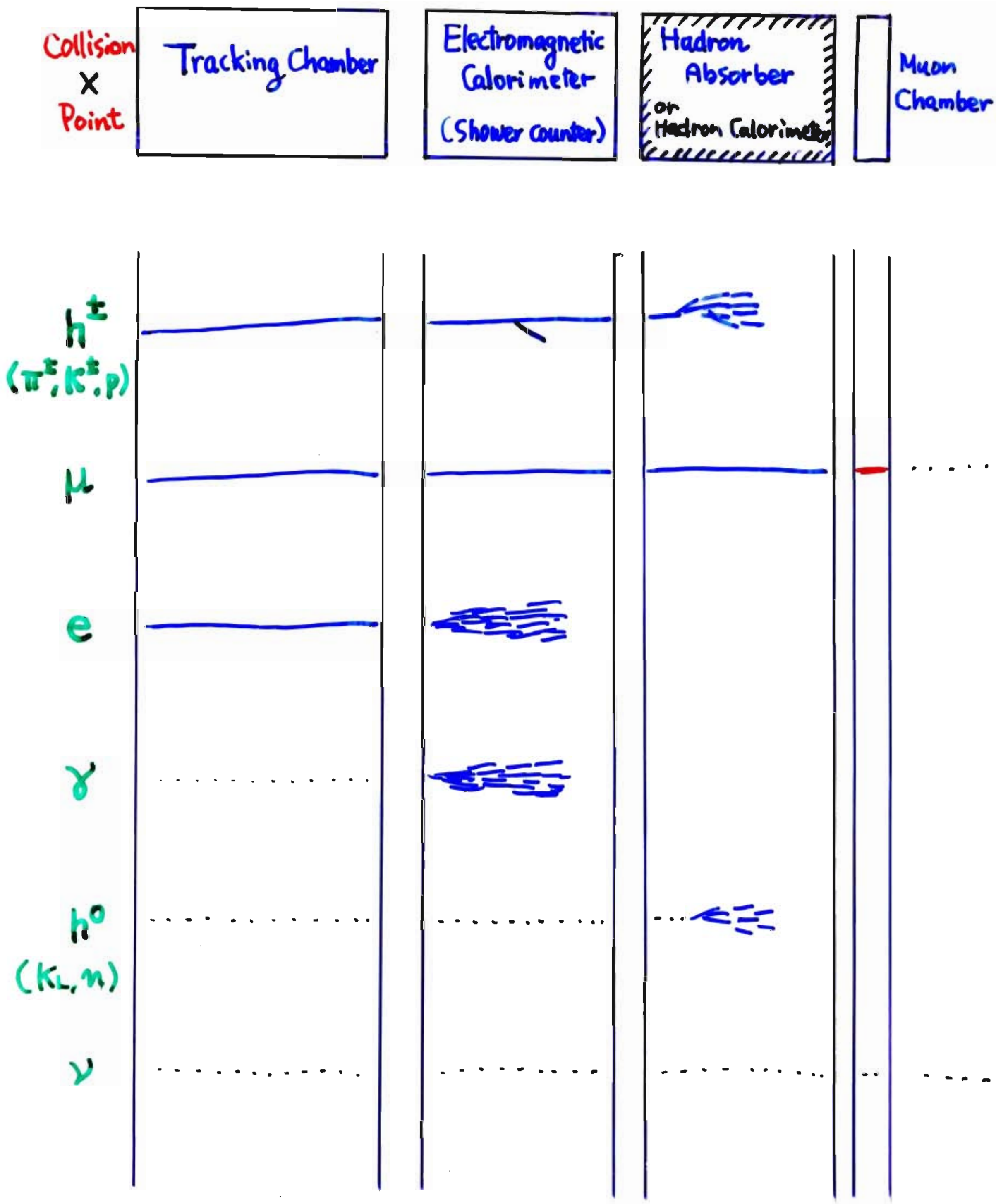
Current and Future Colliders

e^+e^-	location	c.m. energy	luminosity	start	detectors
DAΦNE	Frascati (Italy)	Φ region	$(1\to5)10^{32}$	1997	
BEPC	北京 (中国)	ψ region	10^{31}	1989	BES
CESR	Cornell (USA)	Υ region	$5\ 10^{32}\nearrow$	1979	CLEO
KEK B	つくば	"	10^{34}	1999	BELLE
PEP II	SLAC (USA)	"	$3\ 10^{33}$	1999	BABAR
SLC	SLAC	Z region	10^{30}	1989	SLD
LEP	CERN	"	$2\ 10^{31}$	1989	ALEPH, DELPHI, L3, OPAL
LEP II		171 → 182 GeV →		1996	
xLC	?	500 → TeV region	10^{33}–10^{34}	200?	
$p\bar{p}$					
Tevatron	Fermilab (USA)	1.8 TeV ↗	$2\ 10^{31}\nearrow$	1987	CDF, DØ
LHC	CERN	14 TeV	10^{34}	2004	ATLAS, CMS
$e^{\pm}p$					
HERA	DESY (Germany)	300 GeV (e 27, p 820)	10^{31}	1992	H1, ZEUS

Particle Lifetimes

	γ, e, p, ν	安定	
	n	10^{3} sec	
	μ	2.2×10^{-6} sec	
lifetime ∞	K_L	5.2×10^{-8}	
	$\pi^{\pm}$	2.6×10^{-8}	
	$K^{\pm}$	1.2×10^{-8}	
	Λ	2.6×10^{-10}	$p\pi^-$ 64%
	K_S	0.9×10^{-10}	$\pi^+\pi^-$ 69%
	$\Sigma^+, \Sigma^-, \Xi^0, \Xi^-, \Omega^-$	$(0.8-3) \times 10^{-10}$	
	B	1×10^{-12}	
	charm	$(1-10) \times 10^{-13}$	$D^{*+} \to D^0\pi^+$, $D^0 \to K^-\pi^+$
	τ	3×10^{-13}	
	π^0	0.9×10^{-16}	$\gamma\gamma$ 99%
lifetime 0	η	6×10^{-19}	
	Σ^0	6×10^{-20}	
	J/ψ	1×10^{-20}	e^+e^-, $\mu^+\mu^-$ 7% each
	⋮		
	ρ	4×10^{-24}	
	W, Z	$(2-3) \times 10^{-25}$	

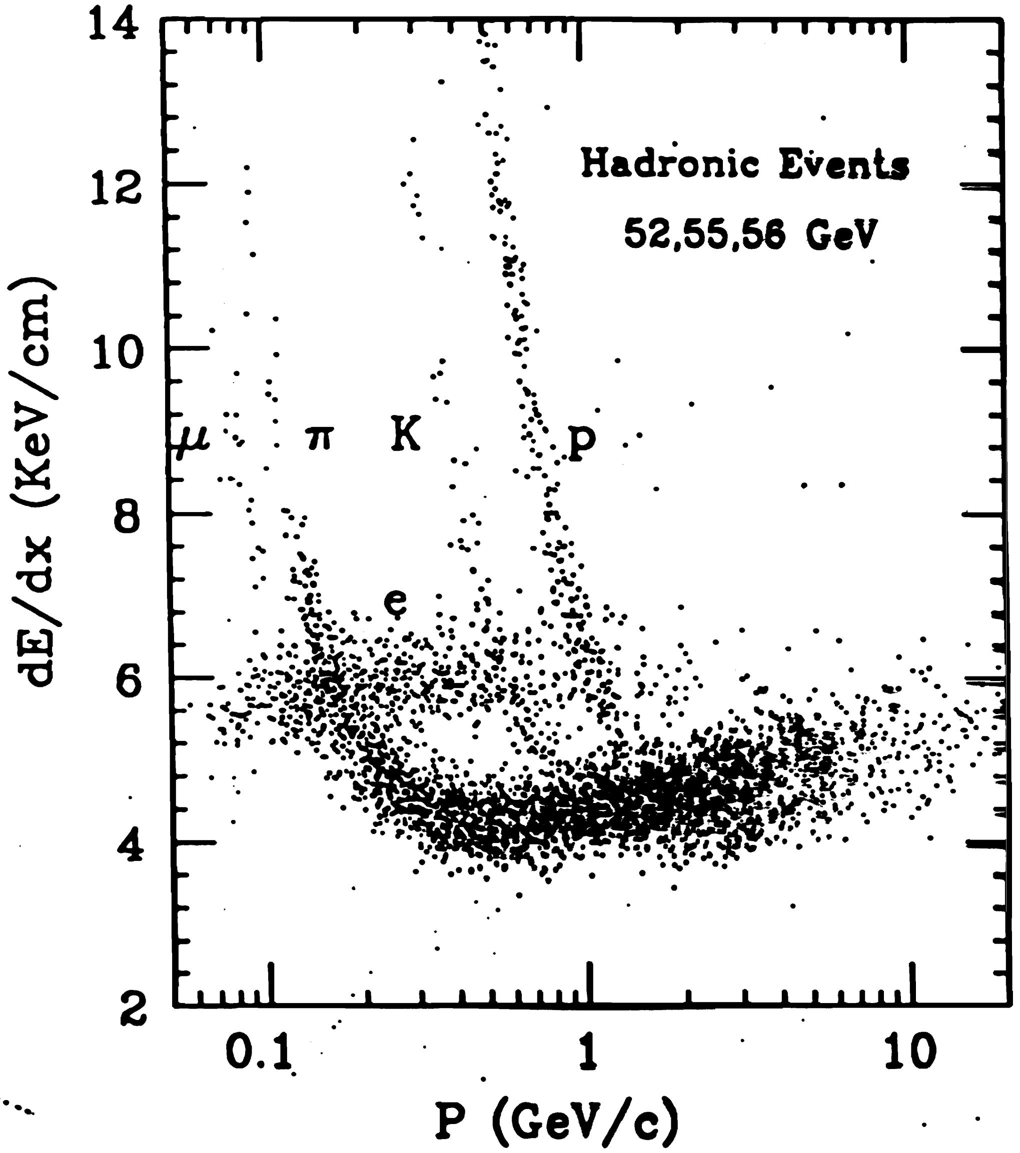
Particle identification

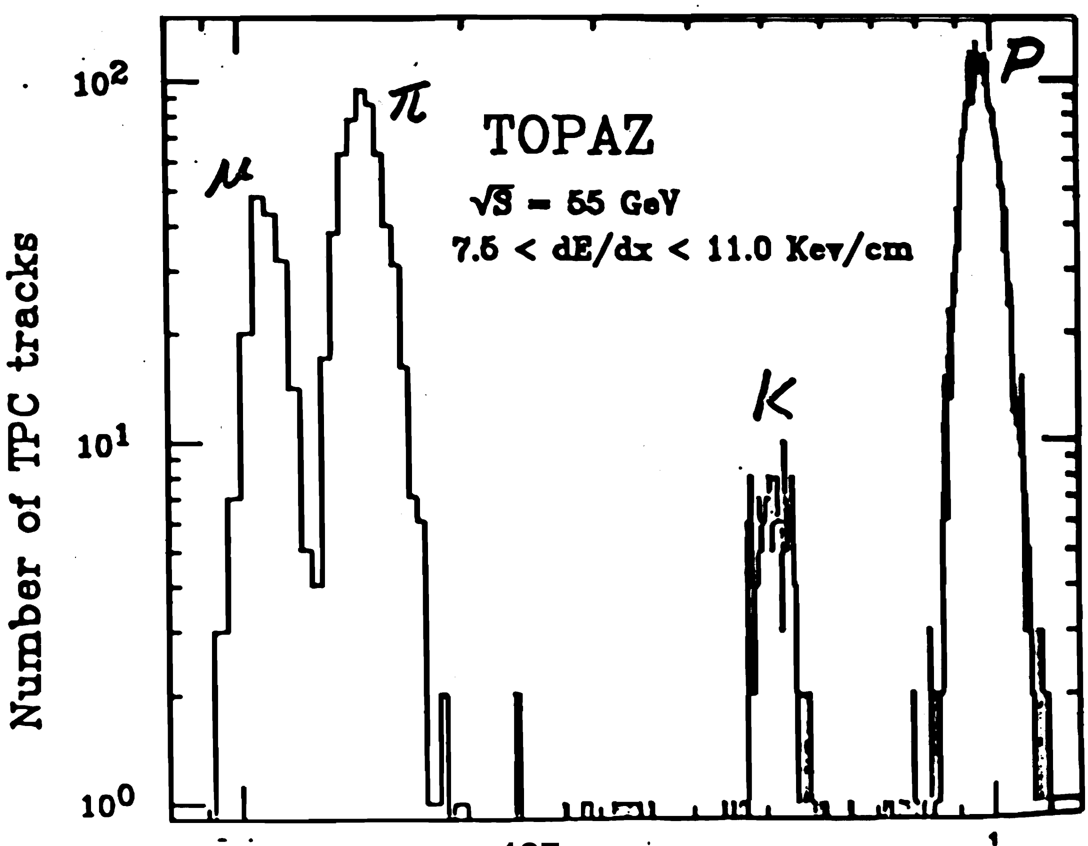


π/K/p separation : measure p and β

- TOF (time of flight)
- $\frac{dE}{dx}$
- Čerenkov

Particle ID by TOPAZ–TPC





Radiative corrections : Where important ?

Electroweak radiative corrections : $O(\alpha)$ in general

But, large corrections appear with a log factor

$$\alpha \log(\text{大きな数})$$

in the following cases

(1) Related to ultraviolet divergence

$$\alpha \log\frac{\Lambda^2}{q^2} \xrightarrow[\text{くりこみ}]{\text{on-shell}} \alpha\left(\log\frac{\Lambda^2}{q^2} - \log\frac{\Lambda^2}{m^2}\right) = -\alpha \log\frac{q^2}{m^2}$$

$\Rightarrow$ use running coupling

(2) Related to soft (infrared) divergence $\leftrightarrow m_\gamma = 0$

$$\alpha \log\frac{s}{E_{min}^2}$$

divergence cancel between real and virtual corrections

No large correction for "total" cross section

(3) Related to collinear singularity $\leftrightarrow m_e \to 0$

(3a) initial radiation

$$\alpha \log\frac{s}{m_e^2}$$

effective reduction of "c.m. energy"

absorbable into "structure function"

(3b) final radiation

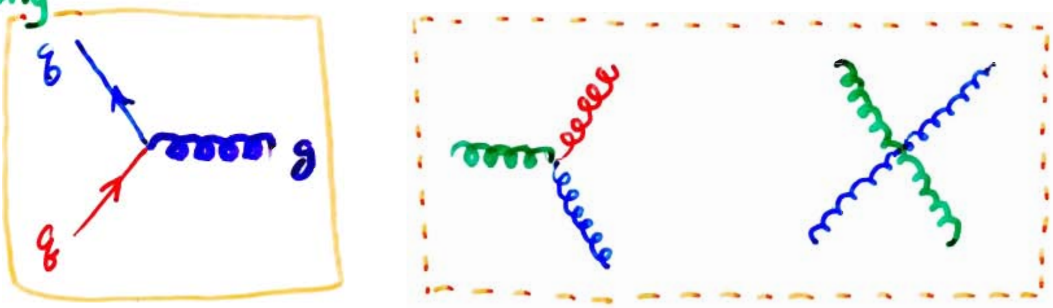
$$\alpha \log\theta_{min}$$

singularity cancel between real and virtual

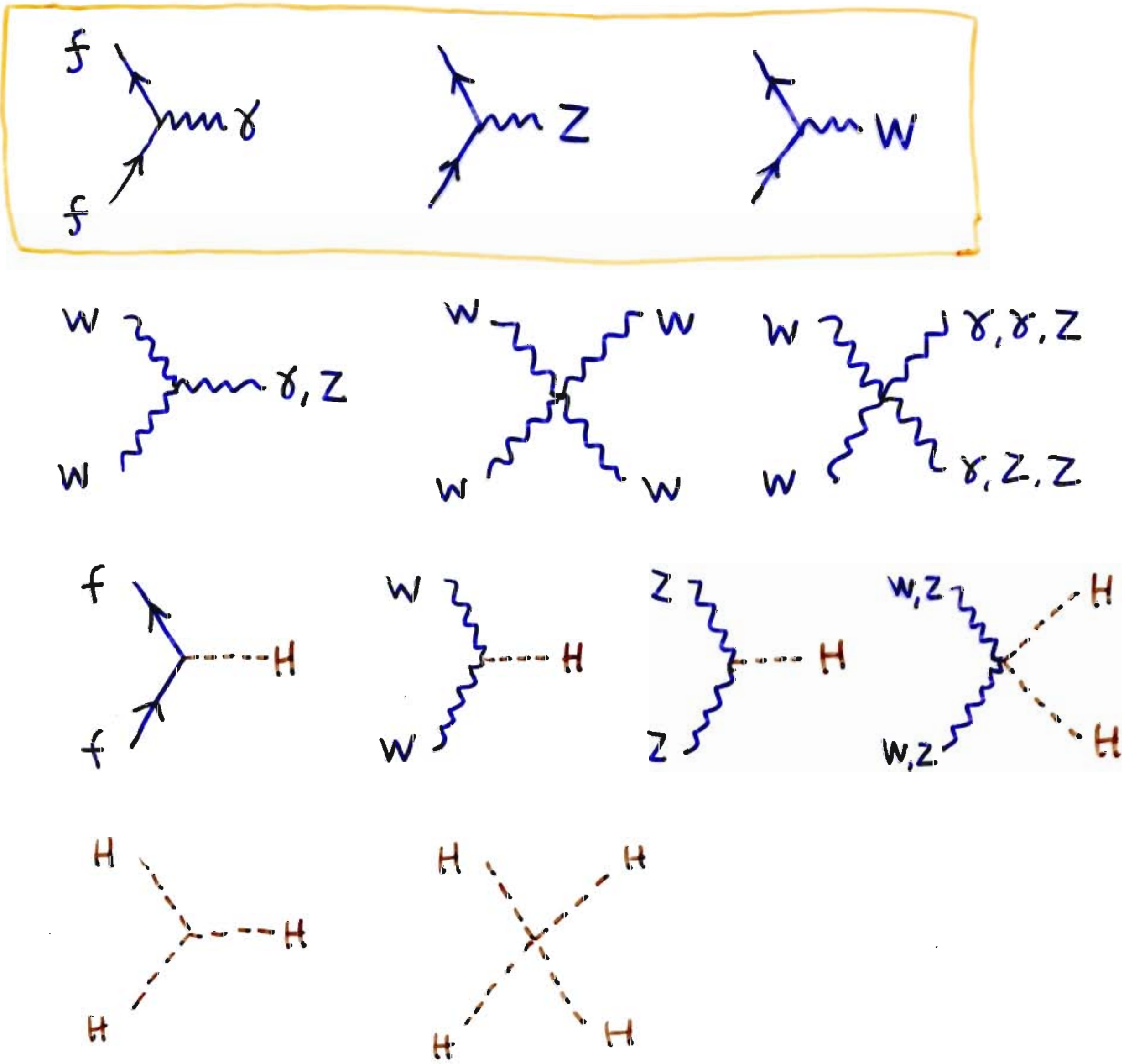
no large correction for "total" cross section

Standard model interactions

Strong



Electroweak



Standard model parameters

1. Basic parameters

Parameters in the fundamental Lagrangian

Should be measured by experiments in principle

		Number	
Gauge couplings	g_s. g. g'	3	
Yukawa couplings	$f^{(d)}$, $f^{(\ell)}$, $h^{(u)}$	13	(3 generations)
Higgs self coupling	λ	1	
Higgs mass term	μ^2	1	

Alternatively one may take

α, G_F, ($\sin^2\theta_W$ or M_Z or M_W)

m_ℓ, m_q, Kobayashi-Maskawa matrix

m_H

Λ_{QCD} (or α_s)

2. Secondary parameters

Calculable in principle, not in practice

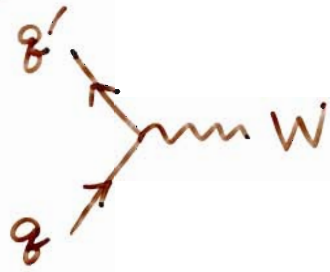
f_π, f_K, ... (decay constant)
hadron masses, magnetic moments etc.

hadron structure functions

various matrix elements

Kobayashi-Maskawa matrix

Charged current interaction of quarks



$$\mathcal{L} = \frac{-g}{2\sqrt{2}} \left(\bar{u}\ \bar{c}\ \bar{t}\right) \gamma_\mu (1-\gamma_5)\, V \begin{pmatrix} d \\ s \\ b \end{pmatrix} \cdot W^{+\mu} + h.c.$$

V : 3×3 unitary matrix

V : 9成分 → quark fields の relative phase は無意味なので $(6-1)=5$ 成分は unphysical

→ 4 physical degrees of freedom

$$\begin{cases} 3 \text{ angles} \\ 1 \text{ phase} \quad (CP\text{-violation}) \end{cases}$$

Measuring KM Matrix

Nuclear β decay
(superallowed Fermi transition)

$|V_{ud}| = 0.9744 \pm 0.0010$

$K \to \pi \ell \nu$
hyperon decay

$|V_{us}| = 0.22$

$\dfrac{b \to u e \nu}{b \to c e \nu}$, $B \to p \bar{p} \pi (\pi)$
(ARGUS)

$0.07 \lesssim \dfrac{|V_{ub}|}{|V_{cb}|} \lesssim 0.2$

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

$\overset{\nu}{\bar{\nu}} N \to$ charm
(CDHS)

$|V_{cd}| \sim |V_{us}|$

$D \to K \ell \nu$
decay rate

$|V_{cs}| \sim 1$

Toponium
single-quark decay ?

B lifetime

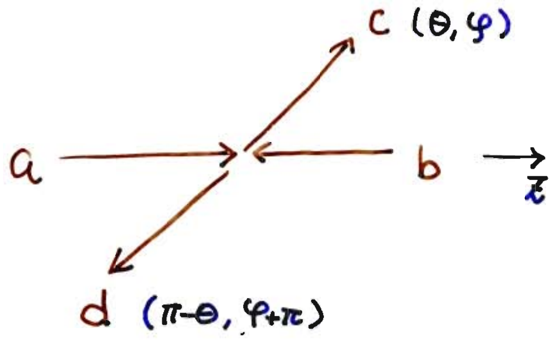
$|V_{cb}| \sim 0.05$

Helicity amplitude

2体 → 2体 反応 $\quad a + b \rightarrow c + d$

helicity $\quad \lambda_a \quad \lambda_b \quad \lambda_c \quad \lambda_d$

重心系



Jacob-Wick formalism : 部分波展開

$$S_{fi} = \delta_{fi} + i(2\pi)^4 \delta^4(P_f - P_i)\, \mathcal{M}_{fi}$$

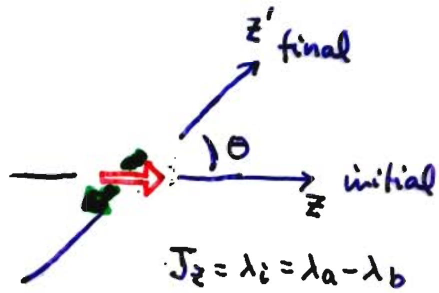
$$\boxed{\mathcal{M}_{fi} = \frac{8\pi}{\sqrt{\bar{\beta}_i \bar{\beta}_f}} \sum_{J=0}^{\infty} (2J+1)\, T^J_{\lambda_a\lambda_b;\lambda_c\lambda_d}(E_{cm})\, d^J_{\lambda_i\lambda_f}(\theta)\, e^{i(\lambda_i-\lambda_f)\varphi}}$$

$$\lambda_i = \lambda_a - \lambda_b$$

$$\lambda_f = \lambda_c - \lambda_d$$

$$\bar{\beta}_i = \frac{2|P_i|}{\sqrt{s}} \qquad \bar{\beta}_f = \frac{2|P_f|}{\sqrt{s}}$$

$d^J_{\lambda_i\lambda_f}(\theta)$: Wigner の d function
(回転行列)



$J_z = \lambda_i = \lambda_a - \lambda_b$

$J_{z'} = \lambda_f = \lambda_c - \lambda_d$

helicity amplitude は

$$\boxed{\mathcal{M}_{fi} = \tilde{\mathcal{M}}_{\lambda_a\lambda_b;\lambda_c\lambda_d}(\sqrt{s}, \cos\theta)\ d^{j_0}_{\lambda_i\lambda_f}(\theta)\, e^{i(\lambda_i-\lambda_f)\varphi}}$$

「最低限の」角分布

と書ける。 $j_0 = \max(|\lambda_i|, |\lambda_f|)$

(但し

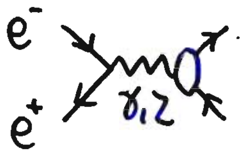
$$\tilde{\mathcal{M}} = \frac{8\pi}{\sqrt{\bar{\beta}_i\bar{\beta}_f}} \sum_{J=j_0}^{\infty} (2J+1)\, T^{J}_{\lambda_a\lambda_b;\lambda_c\lambda_d}(\sqrt{s}) \cdot \frac{d^{J}_{\lambda_i\lambda_f}(\theta)}{d^{j_0}_{\lambda_i\lambda_f}(\theta)}$$

)

$\cos\theta$ の polynomial

s-channel exchange の場合 J は特定の値しかとらない

(例) e^+e^-. one photon exchange
(Z)



ONLY J=1 (at $m_e \to 0$ for Z exch.)

$$\mathcal{M}_{fi} = \tilde{\mathcal{M}}(\sqrt{s})\ d^{1}_{\lambda_i\lambda_f}(\theta)\, e^{i(\lambda_i-\lambda_f)\varphi}$$

d 関数

$$d^j_{mm'}(\theta) = \left[(j+m)!(j-m)!(j+m')!(j-m')!\right]^{1/2}$$

$$\times \sum_k \frac{(-1)^k}{(j-m'-k)!(j+m-k)!(k+m'-m)!k!}$$

$$\times \left(\cos\frac{\theta}{2}\right)^{2j+m-m'-2k} \left(-\sin\frac{\theta}{2}\right)^{m'-m+2k}$$

$$d^j_{m'm}(\theta) = d^j_{mm'}(-\theta)$$

$$d^j_{mm'}(\pi-\theta) = (-1)^{j+m} d^j_{m,-m'}(\theta)$$

$$d^j_{mm'}(0) = \delta_{mm'}$$

$$d^j_{mm'} = (-1)^{m-m'} d^j_{m'm} = (-1)^{m-m'} d^j_{-m,-m'} = d^j_{-m',-m}$$

$$\int_0^\pi d\cos\theta \; d^j_{mm'}(\theta)\, d^{j'}_{mm'}(\theta) = \frac{2}{2j+1}\delta_{jj'}$$

$j_0 \equiv \max(|m|, |m'|)$ とすると

$$d^j_{mm'}(\theta) = d^{j_0}_{mm'}(\theta) \times (\cos\theta \text{の多項式})$$

$d^{1/2}_{\lambda\mu}(\theta)$

λ \ μ	$+\frac{1}{2}$	$-\frac{1}{2}$
$+\frac{1}{2}$	$\cos\frac{\theta}{2}$	$-\sin\frac{\theta}{2}$
$-\frac{1}{2}$	$\sin\frac{\theta}{2}$	$\cos\frac{\theta}{2}$

$d^{1}_{\lambda\mu}$

λ \ μ	$+1$	0	-1
$+1$	$\frac{1+\cos\theta}{2}$	$-\frac{1}{\sqrt{2}}\sin\theta$	$\frac{1-\cos\theta}{2}$
0	$\frac{1}{\sqrt{2}}\sin\theta$	$\cos\theta$	$-\frac{1}{\sqrt{2}}\sin\theta$
-1	$\frac{1-\cos\theta}{2}$	$\frac{1}{\sqrt{2}}\sin\theta$	$\frac{1+\cos\theta}{2}$

$d^{0}_{00} = 1$

$d^{2}_{\lambda\mu}$

λ \ μ	2	1	0	-1	-2
2	$\left(\frac{1+\cos\theta}{2}\right)^2$	$-\frac{1+\cos\theta}{2}\sin\theta$	$\frac{\sqrt{6}}{4}\sin^2\theta$	$-\frac{1-\cos\theta}{2}\sin\theta$	$\left(\frac{1-\cos\theta}{2}\right)^2$
1	$\frac{1+\cos\theta}{2}\sin\theta$	$\frac{1+\cos\theta}{2}(2\cos\theta-1)$	$-\sqrt{\frac{3}{2}}\sin\theta\cos\theta$	$\frac{1-\cos\theta}{2}(2\cos\theta+1)$	$-\frac{1-\cos\theta}{2}\sin\theta$
0	$\frac{\sqrt{6}}{4}\sin^2\theta$	$\sqrt{\frac{3}{2}}\sin\theta\cos\theta$	$\frac{3}{2}\cos^2\theta-\frac{1}{2}$	$-\sqrt{\frac{3}{2}}\sin\theta\cos\theta$	$\frac{\sqrt{6}}{4}\sin^2\theta$
-1	$\frac{1-\cos\theta}{2}\sin\theta$	$\frac{1-\cos\theta}{2}(2\cos\theta+1)$	$\sqrt{\frac{3}{2}}\sin\theta\cos\theta$	$\frac{1+\cos\theta}{2}(2\cos\theta-1)$	$-\frac{1+\cos\theta}{2}\sin\theta$
-2	$\left(\frac{1-\cos\theta}{2}\right)^2$	$\frac{1-\cos\theta}{2}\sin\theta$	$\frac{\sqrt{6}}{4}\sin^2\theta$	$\frac{1+\cos\theta}{2}\sin\theta$	$\left(\frac{1+\cos\theta}{2}\right)^2$

Helicity amplitudes $(\varphi = 0)$

$\boxed{e^- e^+ \to \mu^- \mu^+}$: nonzero amplitudes for $m_e, m_\mu \to 0$

s-γ $\quad \mathcal{M} = -2e^2\, d^1 \qquad |\lambda_i| = |\lambda_f| = 1$

s-Z $\quad -2g_z^2\,(v_e - \lambda_i a_e)(v_\mu - \lambda_f a_\mu)\,\dfrac{s}{s-m_Z^2}\, d^1$

$|\lambda_i| = |\lambda_f| = 1$

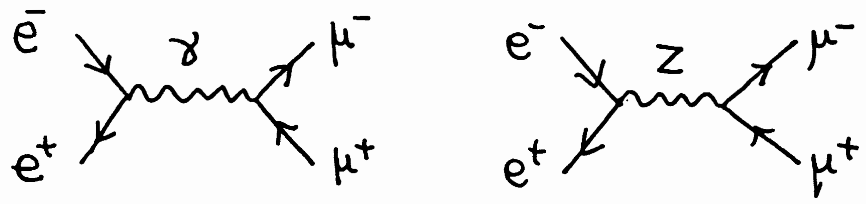
$$g_Z = e/\sin\theta_W \cos\theta_W$$

$$v_e = v_\mu = -\frac{1}{4} + \sin^2\theta_W \quad \ll \quad a_e, a_\mu$$

$$a_e = a_\mu = -\frac{1}{4}$$

$$d^1_{11} = d^1_{-1,-1} = \frac{1}{2}(1+\cos\theta)$$

$$d^1_{1,-1} = d^1_{-1,1} = \frac{1}{2}(1-\cos\theta)$$

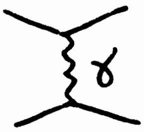


$e^- e^+ \to e^- e^+$ nonzero amplitudes for $m_e \to 0$

s-γ , s-Z : same as for $e^- e^+ \to \mu^- \mu^+$ with $(v_\mu, a_\mu \to v_e, a_e)$

t-γ

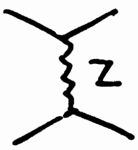
$$\mathcal{M} = \frac{4e^2}{1-\cos\theta}\, d^1 \qquad \lambda_i = \lambda_f = \pm 1 \qquad \begin{array}{cc} h\,\bar{h} & h'\,\bar{h}' \\ (+-) & (+-) \\ (-+) & (-+) \end{array}$$



$$= \frac{4e^2}{1-\cos\theta} \qquad \lambda_i = \lambda_f = 0 \text{ and } h = h' \qquad \begin{array}{cc} (++) & (++) \\ (--) & (--) \end{array}$$

t-Z

$$\mathcal{M} = \frac{4 g_Z^2}{1 + \frac{2m_Z^2}{s} - \cos\theta} (v_e - \lambda_i a_e)^2 d^1 \qquad \lambda_i = \lambda_f = \pm 1$$

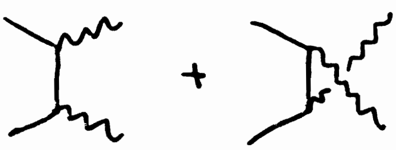


$$= \frac{4 g_Z^2}{1 + \frac{2m_Z^2}{s} - \cos\theta} (v_e^2 - a_e^2) \qquad \lambda_i = \lambda_f = 0 \text{ and } h = h'$$

$e^- e^+ \to \gamma\gamma$

t-e ⊕ u-e

$$\mathcal{M} = -\frac{4e^2}{1-\cos^2\theta}\, d^2 \qquad |\lambda_i| = 1, \quad |\lambda_f| = 2$$



Unitarity and cross section

$$S^\dagger S = 1 \quad (\text{確率の保存})$$

for partial wave amplitudes

elastic channel $(f=i)$ $\quad |T^J| \leq 2$

inelastic channel $\quad |T^J| \leq 1$

Integrated cross section

$$\sigma = \frac{4\pi}{S\bar{\beta}_i^2} \sum_{J=0}^{\infty} (2J+1) |T^J|^2$$

$e^+e^- \xrightarrow[\gamma^*]{} \text{final}$ の場合

$J=1$ only

$T^{J=1} \sim e^2$

$$\Rightarrow \quad \sigma \sim \frac{\alpha^2}{S}$$

Unit of R

$$\sigma(e^+e^- \xrightarrow[\gamma^*]{} \mu^+\mu^-) = \frac{4\pi\alpha^2}{3S}$$