Quantum Energy Teleportation: An Introductory Review

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Abstract

The development of techniques for manipulation of quantum information has opened the door to a variety of protocols for accomplishing unprecedented tasks. In particular, a new protocol of quantum teleportation was proposed in 2008 to achieve effective energy transportation simply via local operations and classical communication —without breaking any known physical laws. This is achieved by extracting zero-point energy of entangled many-body systems by local operations dependent on information obtained by a distant measurement. The concept is reviewed from an introductory viewpoint herein.

1   Introduction

Together with spacetime, matter and information constitute the great building blocks of the Universe. Matter is generally known to comprise the elementary particles such as electrons and photons. But more precisely speaking, the concept encompasses all conserved physical quantities such as energy and electric charge carried by local excitations of elementary-particle fields. Information, however, represents the quantum information carried by matter. All quantum information is consolidated by the quantum state of the fields. A significant essential property of matter is its sameness at a very deep bottom. For example, the properties of an electron at one position are indistinguishable from those of another electron at a different position: they have the same mass, the same spin, and the same electric charge. Quantum mechanics does not distinguish between two particles corresponding to the same field. Then from whence cometh the distinguishing characteristics of individuals and the objects surrounding us? They stem from the full information imprinted on common matter, called the quantum field, that is, the quantum state. In 1982, Wootters and Zurek discovered a remarkable theorem about quantum states [1]. Their theorem shows that general quantum states prohibit their cloning. In contrast to classical information, we cannot make indistinguishable copies of quantum information. In this sense, quantum information is one of the most profound concepts about identity.

Taking into account the above consideration, it is possible to argue that transportation of a quantum state is equivalent to transportation of an object itself with individual characteristics. In quantum mechanics, the transportation of quantum states can be achieved simply by local operations and classical communication (LOCC for short). This protocol was proposed in 1993 and named quantum teleportation [2]. Why is the protocol called 'teleportation'? To illustrate this reason concretely, let us consider the protocol with qubits. Alice and Bob stay at different positions and share a Bell pair of two qubits A and B in a state

$$|E_0\rangle_{AB} = \frac{1}{\sqrt{2}} \left(|+\rangle_A |+\rangle_B + |-\rangle_A |-\rangle_B \right),$$

where $|+\rangle$ ($|-\rangle$) is the up (down) state of the third Pauli operator $\sigma_3 (= |+\rangle\langle+| - |-\rangle\langle-|)$. Alice also has another qubit A' in a unknown state $|\psi\rangle$.

The state of these three qubits can be calculated as

$$|\psi\rangle_{A'} \otimes |E_0\rangle_{AB} = \frac{1}{2} \sum_{\alpha=0}^{3} |E_\alpha\rangle_{A'A} \otimes \sigma_{\alpha B} |\psi\rangle_B, \qquad (1)$$

where the orthogonal Bell states $|E_\alpha\rangle_{A'A}$ are given by $\sigma_{\alpha A'} |E_0\rangle_{A'A}$, and $\sigma_{\alpha A'}$ ($\sigma_{\alpha B}$) is the Pauli operator σ_α for A' (B) with its 0-th component $\sigma_0 = I$. At time $t = t_m$, Alice performs a Bell measurement to identify which $|E_\alpha\rangle_{A'A}$ is realized for the composite system of A' and A. The output is two-bit information of $\alpha(= 0, 1, 2, 3)$. Each emergence probability of α is the same and equal to $1/4$. Because of so-called wavefunction collapse in quantum measurement, the system in the state $|\psi\rangle_{A'} \otimes |E_0\rangle_{AB}$ jumps instantaneously into a different state $|E_\alpha\rangle_{A'A} \otimes \sigma_{\alpha B} |\psi\rangle_B$ corresponding to the measurement result α. Very surprisingly, the state of B becomes a pure state $\sigma_\alpha |\psi\rangle$ and acquires non-trivial dependence on the input state $|\psi\rangle$. This means that B suddenly gets information about $|\psi\rangle$ at the moment of a distant measurement by Alice. In this sense, the quantum information is 'teleported' from Alice to Bob at $t = t_m$. The instantaneous state change at $t = t_m$ is depicted in figure 1.
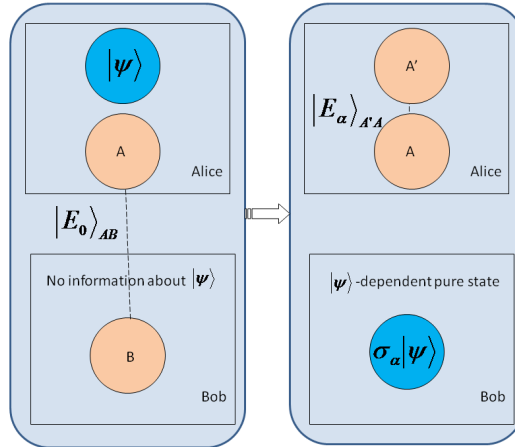

Figure 1: Instantaneous state change after local measurement by Alice in the conventional quantum teleportation protocol with qubits.

After the measurement, Alice announces the result α to Bob via a classical channel such as a telephone. Though the post-measurement state $\sigma_\alpha |\psi\rangle$ of B is different from the original state $|\psi\rangle$, Bob can transform it to the correct

one by locally performing a unitary operation σ_α^{-1} on B at the arrival time $t = t_{op}$ of the information about α. Figure 2 presents a spacetime diagram of the protocol.
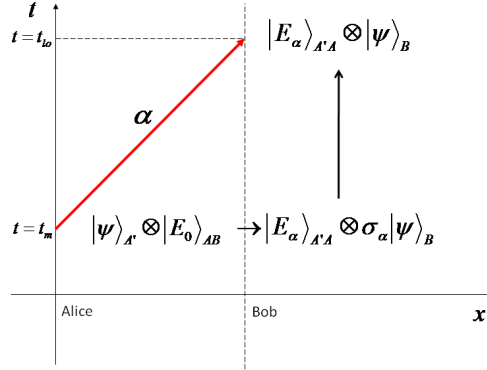

Figure 2: Spacetime diagram of quantum teleportation with qubits.

In order to avoid possible misunderstanding of the concept of teleportation in physics, a comment must be added here: Bob only knows which state of $\sigma_\alpha |\psi\rangle$ ($\alpha = 0 \sim 3$) is realized for B after receiving information about the measurement from Alice. The speed of this classical communication between them never exceeds the speed of light and, therefore, causality is strictly retained.

The protocol of quantum teleportation [2] is really interesting. However, it is not sufficient to teleport energy by itself. Transfer of an excited state to a distant point requires preparation in advance of the same amount of energy of the state at the point. If we do not have enough energy around the distant point, the protocol never works. For example, let us imagine that Alice sends to Bob the spin-up state of σ_3 of a qubit in an external uniform magnetic field parallel to the z axis. For the teleportation, they must share two qubits in a Bell state. The Hamiltonian of each qubit is given by $H_b = b\sigma_3$ with a positive constant b. Note that, in the Bell state, Bob's qubit has zero energy on average. After the state teleportation, the energy of Bob's qubit increases to b on average because the teleported state is the up state. Because Bob's operation in the protocol is local, it is clear that b of the averaged energy must be provided by an external operation device of Bob with a battery, for instance, to drive it. During one round of the protocol, the energy of the

battery decreases by b on average. If Bob does not have energy source like this battery, the up-state teleportation does not succeed. On the other hand, if the down state is teleported to Bob, Bob's qubit loses b of energy on average during his operation. Then the operation device receives b of the averaged energy as a work done by his qubit. Thus the down-state teleportation may be accomplished even if Bob does not have external energy sources to drive the operation device. However, it should be noticed that the averaged energy gain b was originally available for Bob without using the teleportation. Before the operation, Bob's qubit was *already excited* in a Bell state storing b of energy, on average, larger than that of the spin-down ground state. Bob's qubit merely has disgorged the surplus energy due to the transition into the ground state. Therefore, in this protocol, available energy for Bob moves around the region of Bob *without any increase of its total amount.* No energy is teleported in this case. Then do the known laws of physics truly allow energy teleportation? Can we teleport an object with energy to a zero-energy local-vacuum region? Amazingly, the answer is yes—in principle. Energy can be effectively transported simply using local operations and classical communication, just like in the usual quantum teleportation protocol. In quantum mechanics, we can generate quantum states containing a spatial region with negative energy density of quantum fields [3]. Thus, even if we have zero energy in a region where an object is going to be teleported, its energy can be extracted from the vacuum fluctuation of quantum fields, generating negative energy density around there. This can be attained by using a local squeezing operation dependent on the result of a measurement at the starting point of the teleportation. Of course, local energy conservation and all the other physical laws are not violated in the energy teleportation. The protocols, called quantum energy teleportation (QET for short), were first proposed by this author in 2008. QET can be implemented, at least theoretically, to various physical systems, including spin chains [4]-[6], cold trapped ions [7], harmonic chains [8], and quantum fields [9]-[11]. Besides, it has been recently presented that QET would be experimentally verified by using quantum Hall edge currents [12]. Herein, we reviewed the QET protocols from an introductory viewpoint.

The QET mechanism has various links to other research fields in physics. First of all, future QET technology is expected to achieve rapid energy distribution without thermal decoherence inside quantum devices. Because it is not energy but classical information that is sent to the distant point, no heat is generated in the energy transport channel during the short time period

of QET protocols. This aspect will assist in the development of quantum computers. QET also has a close relation to a local-cooling problem, as is explained in section 4. A measurement on a subsystem of a ground-state many-body system often breaks entanglement among the subsystems. This measurement process is accompanied by energy injection to the system because the post-measurement state is not the ground state but instead an excited state. Here we are able to pose an interesting question: Soon after the energy injection, is it possible to extract all the injected energy using only local operations on the measured subsystem? The answer is no, because, from information theory, residual energy is unavoidable for this local-cooling process [4]. The residual energy is lower bounded by the total amount of energy that can be teleported to other subsystems by using the measurement information. The quantum local cooling and QET expose a new aspect of quantum Maxwell's demon [13], who 'watches' quantum fluctuations in the ground state. The amount of teleported energy depends nontrivially on entanglement in the ground state of a many-body system. Hence, QET analyses are also expected to shed new light on complicated entanglement in condensed matter physics and to deepen our understanding of phase transition at zero temperature, which has been recently discussed using the entanglement [14].
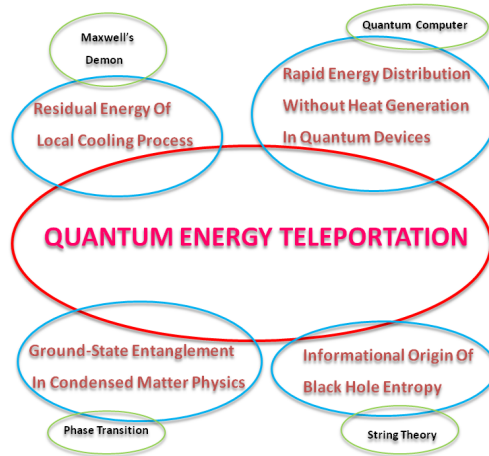

Figure 3: Links between QET and various research fields are schematically summarized.

Moreover, QET provides a new method extracting energy from black holes [11]: Outside a black hole, we perform a measurement of quantum fields and obtain information about the quantum fluctuation. Then positive-energy

wave packets of the fields are generated during the measurement and fall into the black hole. Even after absorption of the wave packets by the black hole, we can retrieve a part of the absorbed energy outside the horizon by using QET. This energy extraction yields a decrease in the horizon area, which is proportional to the entropy of the black hole. However, if we accidentally lose the measurement information, we cannot extract energy anymore. The black-hole entropy is unable to decrease. Therefore, the obtained measurement information has a very close connection with the black hole entropy. This line of argument is expected to lead to further understanding of the origin of black hole entropy, which is often discussed in string theory [15]. Figure 3 presents a schematic summary of the links between QET and these other fields.

The present review is organized as follows: Section 2 presents an elementary description of the QET mechanism to allow the reader to capture the essence of the concept. Section 3 then introduces the most simple example of QET. In section 4, the general theory of QET is constructed for one-dimensional discrete chain models. In section 5, QET with a relativistic quantum field in one dimension is analyzed. The summary and some comments are provided in the last section.

2   Capturing the Essence of QET Mechanism

In this section, an elementary intuitive explanation of QET is presented to allow the reader to capture the essence of the mechanism. More rigorous analyses follow in the later sections. From an operational viewpoint, QET appears to be a kind of scientific magic trick. Let us first imagine a magic trick using two separate empty boxes A and B (figure 4) performed by Alice and Bob. Alice injects some amount of energy to A. Then a secret trick begins to work inside A (figure 5). After a brief while, a magic spell, *abracadabra*, which is a bit number (0 or 1) in this case, is outputted from A. Then Alice announces this information to Bob in front of B (figure 6) and Bob inputs the *abracadabra* to B. B begins some internal process and finally disgorges energy, even though B contained *nothing* at first (figure 7).
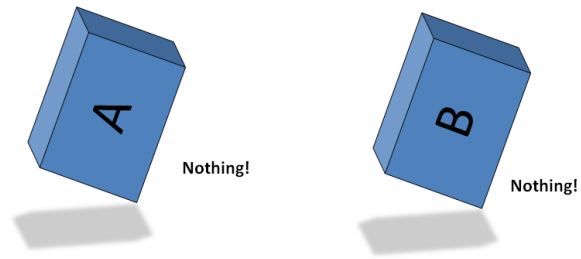
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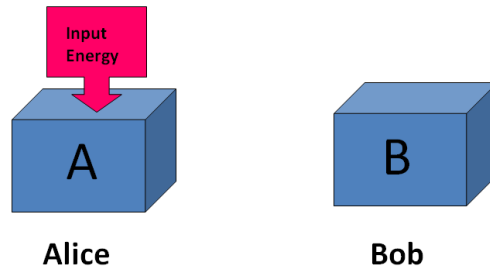Figure 4: Empty boxes A and B for the QET magic trick.

Figure 5: First step of the QET magic trick. Some energy is inputted into the empty box A.
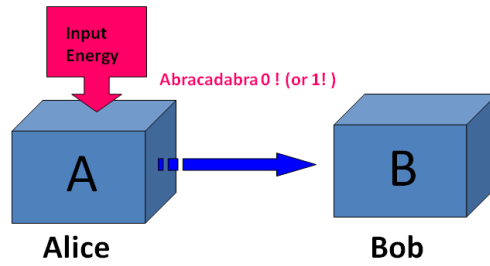
Figure 6: Second step of the QET magic trick, the abracadabra spell, which is a sequence of 0's and 1's, is outputted from A. Then Alice announces it to Bob who is in front of B.
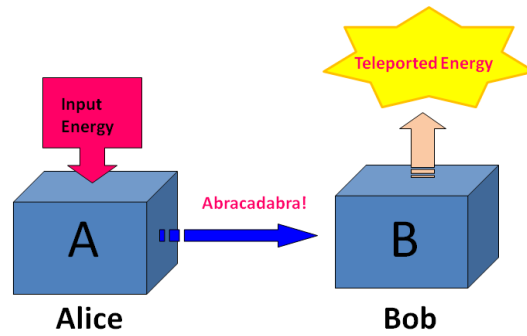

Figure 7: Third step of the QET magic trick. Bob inputs the abracadabra spell to B, which then begins to undergo an internal process that finally disgorges energy, even though B contained nothing at first.

Certainly this looks like energy teleportation. QET is able to achieve this magic using quantum systems.

8

Nothing In Physics

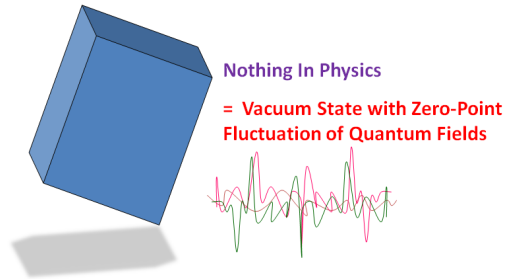**= Vacuum State with Zero-Point Fluctuation of Quantum Fields**

Figure 8: Nothing in quantum theory means the ground state with nonvanishing   quantum fluctuation. This is the key to the QET magic trick.

Then what is the point of this QET magic? The point is related to a question: What is *nothing*? In quantum theory, nothing means the ground state of the system, that is, the eigenstate of total Hamiltonian corresponding to the minimum eigenvalue. For quantum fields in particular, nothing means the vacuum state. It is a very surprising fact of quantum mechanics that nonvanishing zero-point fluctuations exist even in the vacuum state as *nothing* (figure 8).

What is zero-point fluctuation? In classical mechanics, a physical system is completely frozen with no motion possible in its minimum energy state. For example, let us consider a harmonic oscillator composed of a pendulous spring attached to a ball of mass m (figure 9).
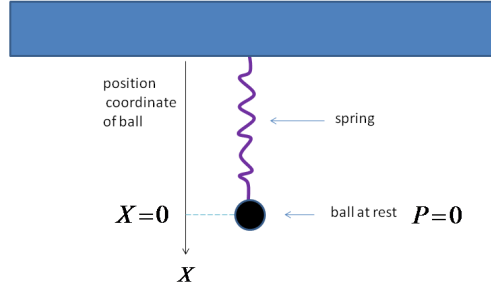
Assuming that the spring force at a position X is given by $F = -m\omega^2 X$ with angular frequency ω of simple harmonic oscillation, its Hamiltonian is given by

$$H = \frac{1}{2m}P^2 + \frac{1}{2}m\omega^2 X^2.$$

In classical theory, both the position X and momentum P of the ball take definite values. In figure 9, the ball is at rest at position $X = 0$ and momentum $P = 0$ in the minimum energy state. However, in quantum theory, position and momentum cannot simultaneously be fixed to arbitrary precision even in the ground state according to Heisenberg's uncertainty principle: $\Delta X \Delta P \geq \hbar/2$. Here, ΔX (ΔP) is the quantum uncertainty in the position (momentum) of the ball. Zero-point fluctuation is the random motion induced by this quantum uncertainty. The situation is depicted in figure 10.
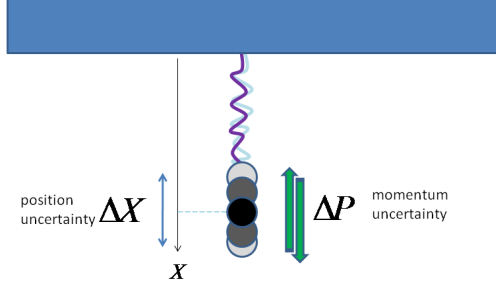
The minimum energy E_g can be roughly estimated by minimization of

$$E_g = \frac{1}{2m}\Delta P^2 + \frac{1}{2}m\omega^2\Delta X^2$$

with $\Delta X \Delta P = \hbar/2$. This yields the following estimation:

$$E_g = \frac{1}{2}\hbar\omega. \tag{2}$$

It is well known that rigorous derivation of the ground-state energy also gives the same result in Eq. (2). The ground-state energy is called the zero-point energy. This simple example exposes that zero-point fluctuation is capable of carrying nonzero energy. And it is not only the harmonic oscillator but also other general interacting many-body systems that have zero-point fluctuation with nonvanishing energy in the ground state. In figure 11, zero-point fluctuation of coupled harmonic oscillators is schematically depicted as an example.
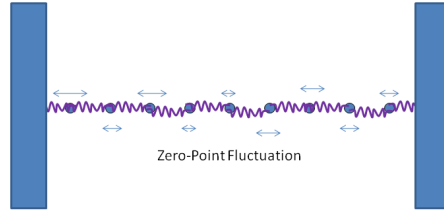
Figure 11: Zero-point fluctuation of coupled harmonic oscillators is schematically depicted. Each subsystem fluctuates with nonzero energy density even in the ground state.

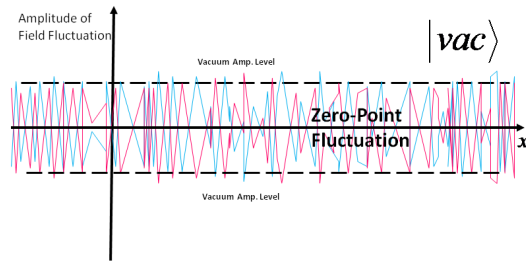Each subsystem is fluctuating with nonzero energy density in the ground state.

Figure 12: Zero-point fluctuation of a boson field in one dimension is schematically depicted. The vertical line implies amplitude of (coarse-grained) field fluctuation. The horizontal line describes spatial coordinate x. Mathematical description of the quantum fluctuation is a superposition of various configuration states. This situation is simplified in the figure and only two different configurations of the fluctuation are exhibited by red and blue broken lines. They fluctuate at a typical amplitude order fixed by the quantum uncertainty relation.

Of course, quantum fields also have zero-point fluctuation in the vacuum state. In this case, the fluctuation is also called vacuum fluctuation. In

figure 12, zero-point fluctuation of a boson field in one dimension is schematically depicted. The vertical line implies amplitude of (coarse-grained) field fluctuation. The horizontal line describes the spatial coordinate x. The mathematical description of the quantum fluctuation is a superposition of various configuration states. This situation is simplified in figure 12 and only two different configurations of the fluctuation are exhibited by the red and blue broken lines. They are fluctuating with a typical amplitude order fixed by the quantum uncertainty relation. In later discussions, let us shift the origin of energy by subtracting the zero-point energy E_g from original energy values so as to make the value of the ground state zero:

$$E' = E - E_g.$$

Such a shift is always allowed without changing physics as long as we do not take into account general relativity. This is because it is not the absolute value but the difference in energies of two states that is physically observable. Therefore, the total energy takes nonnegative values. Regardless of this nonnegativity, quantum theory has a very amazing feature that energy density can take negative values [3].
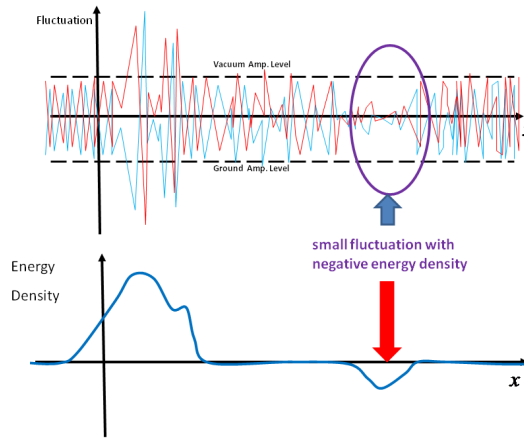
Figure 13: Illustration of emergence of negative energy density. In the upper part of the figure, a typical situation of local squeezing of the fluctuation is schematically depicted. The part surrounded by an ellipse shows the region of suppressed fluctuation with average energy density smaller than that of the vacuum state. In the lower part of the figure, energy density in this region must take a negative value because energy density of the vacuum state is zero and larger than that of the surrounded region.

By superposing total-energy eigenstates, quantum fluctuation in a local region can be more suppressed (squeezed) than that in the vacuum state via a quantum interference effect. In the upper part of figure 13, a typical situation of local squeezing of the fluctuation is schematically depicted. The part surrounded by an ellipse shows the region of suppressed fluctuation with average energy density lower than that of the vacuum state. As seen in the lower part of figure 13, energy density in this region must take a negative value because energy density of the vacuum state is zero and larger than that of the surrounding region. It is worth stressing here that the total energy cannot be negative even though we have a region with negative energy density. This implies that we have other regions with sufficient positive energy to compensate for this negative energy, as in figure 13. Local energy, which is an integral of energy density with an appropriate window function in a compact support, can also take a negative value, i.e., a value smaller than that of the vacuum state (zero).
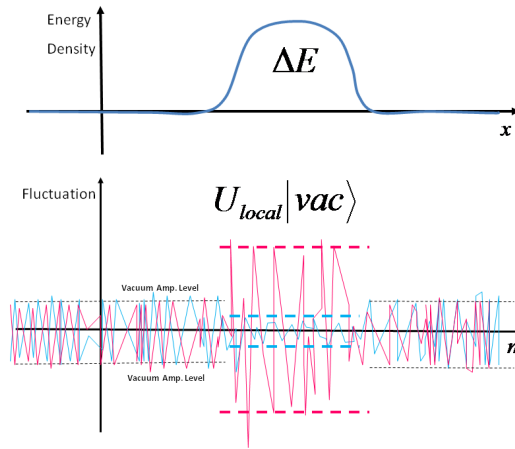
Figure 14: Illustration of passivity of the ground state. If any local unitary operation is performed on the vacuum state, the energy of the field does not decrease but instead increases on average. This is because the state is not the vacuum state but an excited state. In the upper part of the figure, total energy is positive on average. In the lower part of the figure, the operation enhances the amplitude of the red component even though it decreases the amplitude of the blue component.

This fact may tempt us to directly extract zero-point energy from the vacuum, which really carries zero local energy *larger than the negative one*. If this was possible, we would get energy without any cost, but unfortunately,

it is not. If we could extract energy from the vacuum state, the field would be in a state with total energy less than that in the vacuum state, that is, a negative total-energy state. However, the total energy must be nonnegative. Therefore, such energy extraction cannot be attained in physics. For example, if any local unitary operation U_{local} ($\neq I$) is performed in the vacuum state $|vac\rangle$, the energy of the field does not decrease but instead increases. This is because $U_{local}|vac\rangle$ is not the vacuum state but an excited state [16]. Thus, just like in the upper part of figure 14, the expectation value of total energy must be positive:

$$\Delta E = \langle vac|U_{local}^{\dagger}HU_{local}|vac\rangle > 0. \tag{3}$$

Therefore, the operation requires injection of the additional energy ΔE to the field. Here, it should be emphasized that this energy increase takes place just *on average*. It can happen that an operation U_{local} decreases the amplitudes and energy contributions of a few components from among a large number of superposed fluctuation patterns (the blue component, for instance, in the lower part of figure 14). This aspect becomes one of the key points in the construction of QET later. However, if it happens, other components, like the red one in the lower part of figure 14, must be enhanced by much more in their energy contributions to satisfy the average-value relation in Eq. (3). The fundamental property in Eq. (3) is called passivity of the vacuum state.
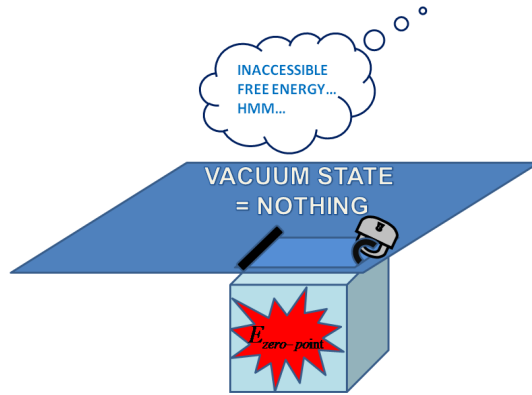

Figure 15: Zero-point energy of the vacuum state may be considered to be inaccessible free energy hidden in a safe underground.

Due to the passivity, one might think that the zero-point energy of the vacuum state is actually inaccessible free energy hidden in a safe underground

(figure 15). It really exists in *nothing*, but cannot be harnessed as long as available operations are local.

Though zero-point energy is totally useless for a single experimenter at a fixed position, it becomes available if two separate experimenters are able to perform both local operations and classical communication—this is QET. In the ground state of an ordinary many-body system, like for a quantum field, there exists a quantum correlation called *entanglement* [21] among zero-point fluctuations of the subsystems. As seen in figure 16, zero-point fluctuations of the vacuum in regions A and B are correlated due to the kinetic term of its Hamiltonian [22].
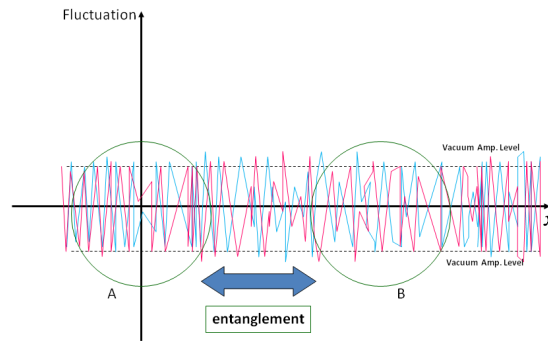
Figure 16: Zero-point fluctuations of a quantum field in regions A and B are entangled.

By virtue of the existence of entanglement, when local zero-point fluctuation is measured at a position, the measurement result includes information about quantum fluctuation at a distant position. This vacuum-state entanglement is at the heart of the QET protocol with quantum fields. As the first step of the protocol, zero-point fluctuation is measured in region A to afford the result α (figure 17).

Figure 17: QET protocol. At first, zero-point fluctuation is measured in region A and a measurement result corresponding to the blue component is obtained. The zero-point fluctuation is locally enhanced by inputting energy. This result includes information about post-measurement quantum fluctuation in region B. The red component vanishes because of the wavefunction collapses on quantum measurement.
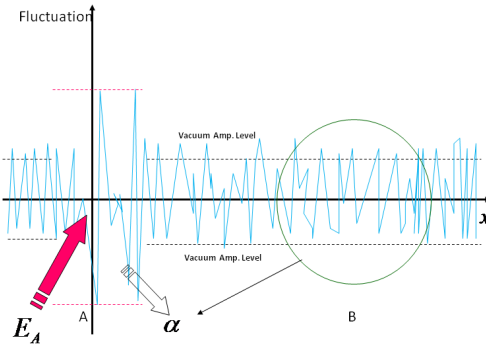
This result α includes information about post-measurement quantum fluctuation in region B via entanglement. Hence, we can estimate the quantum fluctuation at B on the basis of α. In the example shown in figure 17, the value of α corresponding to the blue-line component is obtained by this one-shot measurement. In this case, the other (red-line) component vanishes because of the wavefunction collapse when a quantum measurement is performed. (Actually, practical measurements of local quantum fluctuation are unable to select out a single configuration of fluctuation, as depicted in figure 17. However, it is still true that the measurement results include some information about fluctuation at a distant point, even though the amount of information reduces as the distance increases.) It should be noted that the measurement device injects positive energy E_A into the field during this measurement process because of the vacuum-state passivity. This injected energy is regarded as energy input in the QET protocol. As the second step, the measurement result α is announced from A to B via a classical channel. The speed of this announcement can attain the velocity of light, in principle. During this classical communication, we can neglect the time evolution of the system, as explained later. On the basis of the announced α, we can devise a strategy, that is, a local unitary operation $U_B(\alpha)$ dependent on α, to suppress the realized quantum fluctuation at B for each value of α. As the final step, $U_B(\alpha)$ is performed on the quantum fluctuation of B. This operation

17

yields negative energy density around B (figure 18) by suppressing only the amplitude of one component of fluctuation observed in the measurement.
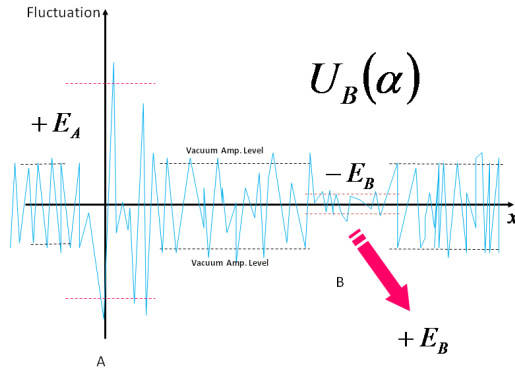
In figure 18, the blue component is suppressed by $U_B(\alpha)$. The operation $U_B(\alpha)$ with the value of α corresponding to the blue component does not need to simultaneously suppress the red component in figure 16 because it has been already been eliminated by this one-shot measurement. This breaks the passivity barrier against harnessing zero-point energy. After the last step, the local energy of the field around B takes a negative value $-E_B$. According to local energy conservation, positive energy $+E_B$ is moved from the field to external systems, including the device executing $U_B(\alpha)$. This is regarded as energy output in the QET protocol and can be harnessed for an arbitrary purpose. Thus, QET really succeeds in effective energy teleportation in an operational sense. After completion of the protocol, the total energy of the field is equal to $E_A - E_B$. Therefore, the input energy E_A is not smaller than the output energy E_B:

$$E_A \geq E_B,$$

because the total energy does not become negative. Note that the positive local energy $+E_A$ of the field in region A compensates for the negative local energy $-E_B$ of the field in region B in late-time evolution. Hence, the late-time evolution with cool-down of the system after one round of a short-acting

18

QET protocol plays a clearing role at the resuming step of the protocol to prepare for the next round.

This QET mechanism can be summarized using an analogy as follows. As seen in figure 15, the zero-point energy, which will become the output energy $+E_B$ of QET, is analogous to the energy $E_{zero-point}$ saved in the locked safe underground. In QET, we get information about a key to the safe, allowing us to extract the zero-point energy by a remote measurement at A via the vacuum-state entanglement (figure 19).
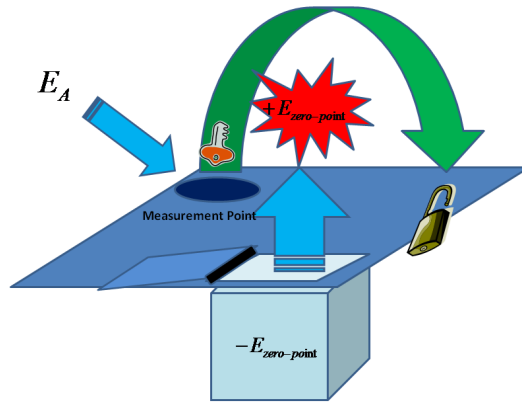
Figure 19: QET is analogous to the following process: We get information, a key to the safe depicted in figure 15, by a remote measurement at A. Using this, the stored zero-point energy can be extracted. However, we must pay the quantum fluctuation at A for this. The cost is positive energy, which is larger than the extracted zero-point energy from the safe at B.

However, we must pay the quantum fluctuation at A for this extraction. The cost is energy E_A, which is larger than the extracted zero-point energy $E_{zero-point}(= E_B)$ taken from the safe at B.

From an operational point of view, the quantum field system can be described as a microscopic 'energy transporter' from Alice to Bob—yes, like the one in Star Trek—as depicted by the two ellipses connected by the dashed line in figure 20.

Figure 20: Instantaneous state change of QET energy transporter. A quantum field system is depicted as an energy transporter by two ellipses connected by the dashed line in the figure. In the left part of the figure, the field is in the vacuum state carrying no available energy. Performing a measurement with energy input to Alice's transporter device, an energetic fluctuation pattern is realized inside Bob's device as depicted in the right part of the figure.
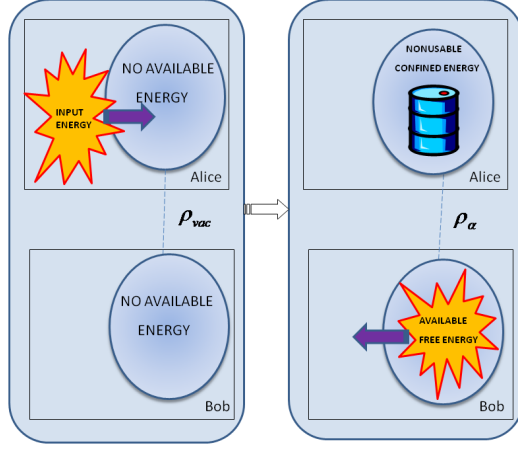
Before Alice's measurement, the field is in the vacuum state $\rho_{vac} = |vac\rangle\langle vac|$ carrying no available energy, as depicted in the left part of figure 20. The zero-point energy of quantum fluctuation has not yet been activated for use. By performing a measurement with energy input to Alice's transporter device, one component of the energetic fluctuation pattern is realized instantaneously inside Bob's device, as depicted in the right part of figure 20. More precisely speaking, the post-measurement state ρ_α of the field, which corresponds to the measurement result α, carries the available zero-point field energy, which can be extracted later by Bob's operation $U_B(\alpha)$ generating negative energy of fluctuation. Note that the energy injected to Alice's device becomes inactive due to the decrease of the vacuum entanglement during her measurement, as will be explained in section 4. Using both this protocol of energy transportation and the standard teleportation protocol for quantum information, it is possible, in principle, to teleport an object with energy to a zero-energy local-vacuum region.

Now, the trick to the QET magic considered first is clear; it is depicted in figure 21.
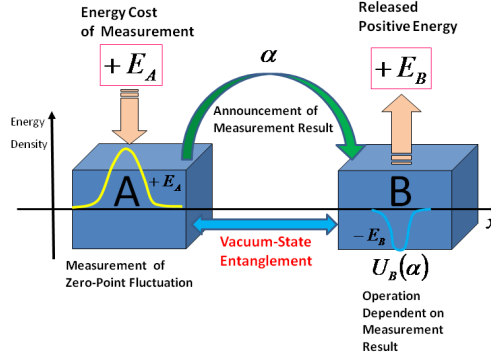
Figure 21: Secrets to the first QET magic trick.

The trick inside A in figure 6 is the measurement of zero-point fluctuation. The input energy E_A is consumed when performing the measurement. The abracadabra announced from Alice to Bob is the measurement result α. The process inside B in figure 7 is the local operation $U_B(\alpha)$ dependent on α. After the completion of this magic, positive local energy $+E_A$ has been hidden inside A and negative local energy $-E_B$ inside B.

In this section, we have omitted the time evolution of quantum fields during the QET protocol for two reasons: The first is that the field system can be nonrelativistic. In condensed matter physics, we have many systems, including quantum Hall edge current [17] [12] and graphene [18], that are described by effective field theory. The speed of energy carriers in these systems is much smaller than the velocity of light. Therefore, the local operations and classical communication of QET can be assumed to take a very short time, during which we may neglect time evolution of the effective fields. The second reason is that a QET process in which time evolution is essentially irrelevant can be actually constructed using relativistic fields [11]. We are able to consider a setting in which wave packets excited by Alice's measurement do not propagate toward Bob in time evolution. Thus, Alice can send only information to Bob, avoiding directly sending Bob the energy emitted from the measurement device. This allows us to establish a nontrivial QET protocol between them. In fact, such a QET protocol with a relativistic field is introduced in section 5.

Before closing this section, it is worth stressing that we do not need to worry about time-energy uncertainty relations for measurements of the out-

put energy E_B of QET. In contrast to the position-momentum uncertainty relation, time-energy uncertainty relations are not fundamental laws. They are conceptually dependent on purposes of our tasks and models of energy measurements, and do not have a universal meaning in quantum theory [19]. This is essentially because time in quantum theory is not an observable to be measured but an external parameter. We must fix in advance a definite time slice in our spacetime to define a quantum state of a system and perform measurements of arbitrary observables at the time. The Hamiltonian, which stands for the total energy of the system, is just one of ordinary observables we can measure instantaneously at a fixed time slice. In fact, the very famous pointer-basis measurement proposed by von Neumann [20] is capable of attaining an instantaneous measurement of energy as follows. Let us consider a system S with Hamiltomian H_S and a probe system to measure energy of S. The probe is a quantum particle in an infinite line parametrized by a spatial coordinate x. The position x of the particle is interpreted as the position of the energy pointer of the measurement device. Let us assume that the initial state of the pointer particle is localized at $x = 0$ as

$$\psi_P(x) = \delta(x).$$

The pointer particle has no free Hamiltonian, but couples with S via a measurement interaction given by

$$H_m(t) = \delta(t) H_S \otimes (-i\hbar \partial_x). \tag{4}$$

Let us prepare a state of S such that

$$|\Psi_S\rangle = \sum_n c_n |E_n\rangle,$$

where c_n are complex coefficients, and $|E_n\rangle$ is an eigenstate of H_S corresponding to an eignvalue E_n. After the instantaneous interaction (4), the state of the composite system becomes

$$\exp\left[-H_S \otimes \partial_x\right] (|\Psi_S\rangle \otimes \psi_P(x)) = \sum_n c_n |E_n\rangle \otimes \delta(x - E_n).$$

Soon after the switch off of the interaction, we can perform a projective measurement of the position x of the pointer particle. This gives a value of energy of S at $t = 0$ as a single-shot measurement result. Therefore

energy can be measured instantaneously. Meanwhile, if one may consider a bad class of energy measurements, time-energy uncertainty relations hold and prevent us from measuring energy precisely. For example, tracking time evolution of a state of a system during a time T allows us to estimate energy of the system by using time-Fourier transformation of the state. However, the estimation has inevitable error of order of $\hbar/T$ in a similar way to the momentum measurement of a particle confined in a finite spatial region. In our QET analysis, we do not adopt such bad measurements governed by (non-universal) time-energy uncertainty relations. In the reference [12] and the last part of the next section, more realistic models of extraction and measurement of E_B are discussed.

3   Minimal QET Model

In this section, the most simple example of QET is reviewed. We adopt the natural unit $\hbar = 1$. For a detailed analysis, see [6]. The system consists of two qubits A and B. Its Hamiltonian reads

$$H = H_A + H_B + V,$$

where each contribution is given by

$$H_A = h\sigma_A^z + \frac{h^2}{\sqrt{h^2 + k^2}}, \tag{5}$$

$$H_B = h\sigma_B^z + \frac{h^2}{\sqrt{h^2 + k^2}}, \tag{6}$$

$$V = 2k\sigma_A^x\sigma_B^x + \frac{2k^2}{\sqrt{h^2 + k^2}}, \tag{7}$$

and h and k are positive constants with energy dimensions, σ_A^x (σ_B^x) is the x-component of the Pauli operators for the qubit A (B), and σ_A^z (σ_B^z) is the z-component for the qubit A (B). The constant terms in Eqs. (5)–(7) are added in order to make the expectation value of each operator zero for the ground state $|g\rangle$:

$$\langle g|H_A|g\rangle = \langle g|H_B|g\rangle = \langle g|V|g\rangle = 0.$$

Because the lowest eigenvalue of the total Hamiltonian H is zero, H is a nonnegative operator:

$$H \geq 0,$$

which means that expectation value of H for an arbitrary state $|\Psi\rangle$ is non-negative:

$$\langle\Psi|H|\Psi\rangle \geq 0.$$

Meanwhile, it should be noted that H_B and H_B+V have negative eigenvalues, which can yield negative energy density at B. The ground state is given by

$$|g\rangle = \frac{1}{\sqrt{2}}\sqrt{1 - \frac{h}{\sqrt{h^2 + k^2}}}|+\rangle_A|+\rangle_B$$

$$-\frac{1}{\sqrt{2}}\sqrt{1 + \frac{h}{\sqrt{h^2 + k^2}}}|-\rangle_A|-\rangle_B,$$

where $|\pm\rangle_A$ ($|\pm\rangle_B$) is the eigenstate of σ_A^z (σ_B^z) with eigenvalue ± 1. A QET protocol is constructed by the following three steps:

- I. A projective measurement of observable σ_A^x is performed on A in the ground state $|g\rangle$ and a measurement result $\alpha = \pm 1$ is obtained. During the measurement, a positive amount of energy

$$E_A = \frac{h^2}{\sqrt{h^2 + k^2}} \tag{8}$$

  is injected to A on average.

- II. The result α is announced to B via a classical channel at a speed much faster than the velocity of energy diffusion of the system.

- III. Let us consider a local unitary operation on B depending on the value of α given by

$$U_B(\alpha) = I_B \cos\theta - i\alpha\sigma_B^y \sin\theta,$$

  where θ is a real constant that satisfies

$$\cos(2\theta) = \frac{h^2 + 2k^2}{\sqrt{(h^2 + 2k^2)^2 + h^2 k^2}}, \tag{9}$$

$$\sin(2\theta) = \frac{hk}{\sqrt{(h^2 + 2k^2)^2 + h^2 k^2}}. \tag{10}$$

$U_B(\alpha)$ is performed on B. During the operation, a positive amount of energy

$$E_B = \frac{h^2 + 2k^2}{\sqrt{h^2 + k^2}} \left[\sqrt{1 + \frac{h^2 k^2}{(h^2 + 2k^2)^2}} - 1 \right] \tag{11}$$

is extracted from B on average.

The protocol is schematically depicted in figure 22.
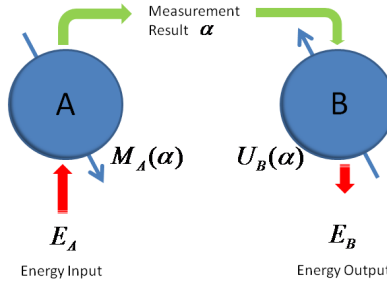
Figure 22: Minimal model of QET with two qubits.

Firstly, the projection operator corresponding to each measurement result α of σ_A^x is given by

$$P_A(\alpha) = \frac{1}{2} \left(1 + \alpha \sigma_A^x \right).$$

The post-measurement state of the two qubits with output α is given by

$$|A(\alpha)\rangle = \frac{1}{\sqrt{p_A(\alpha)}} P_A(\alpha)|g\rangle,$$

where $p_A(\alpha)$ is the emergence probability of α for the ground state and given by $\langle g|P_A(\alpha)|g\rangle$. It is easy to check that the average post-measurement state given by

$$\sum_\alpha p_A(\alpha)|A(\alpha)\rangle\langle A(\alpha)| = \sum_\alpha P_A(\alpha)|g\rangle\langle g|P_A(\alpha)$$

25

has a positive expectation value E_A of H, which has an energy distribution localized at A. In fact, the value defined by

$$E_A = \sum_\alpha \langle g|P_A(\alpha)HP_A(\alpha)|g\rangle = \sum_\alpha \langle g|P_A(\alpha)H_AP_A(\alpha)|g\rangle \qquad (12)$$

can be computed straightforwardly and Eq.(8) is obtained. This injected energy E_A is regarded as the QET energy input via the measurement of A. During the measurement, E_A is transferred from external systems, including the measurement device and its power source, respecting local energy conservation.

The nontrivial feature of this measurement is that it does not increase the average energy of B at all. By explicit calculations using

$$[\sigma_A^x, H_B] = [\sigma_A^x, V] = 0,$$

the average values of H_B and V are found to remain zero after the measurement and are the same as those in the ground state:

$$\sum_\alpha \langle g|P_A(\alpha)H_BP_A(\alpha)|g\rangle = \langle g|H_B|g\rangle = 0,$$

$$\sum_\alpha \langle g|P_A(\alpha)VP_A(\alpha)|g\rangle = \langle g|V|g\rangle = 0.$$

Thus we cannot extract energy from B by local operations *independent* of α soon after the measurement. To verify this fact explicitly, let us consider a local unitary operation W_B independent of α and performed on B. Then the post-operation state ω is given by

$$\omega = \sum_\alpha W_B P_A(\alpha)|g\rangle\langle g|P_A(\alpha)W_B^\dagger$$

$$= W_B \left(\sum_\alpha P_A(\alpha)|g\rangle\langle g|P_A(\alpha) \right) W_B^\dagger.$$

The energy difference after the operation is calculated as

$$E_A - \mathrm{Tr}\,[\omega H] = -\langle g|W_B^\dagger \left(H_B + V \right) W_B|g\rangle, \qquad (13)$$

where we have used

$$W_B^\dagger H_A W_B = H_A W_B^\dagger W_B = H_A,$$

$$\left[W_B^\dagger\left(H_B + V\right)W_B, \ P_A(\alpha)\right] = 0,$$

and the completeness relation of $P_A(\alpha)$:

$$\sum_\alpha P_A(\alpha) = 1_A.$$

From Eq. (13), it is proven that the energy difference is not positive:

$$E_A - \mathrm{Tr}\left[\omega H\right] = -\langle g|W_B^\dagger H W_B|g\rangle \leq 0,$$

because of a relation such that

$$\langle g|W_B^\dagger H_A W_B|g\rangle = \langle g|H_A|g\rangle = 0$$

and the nonnegativity of H. Therefore, as a natural result, no local operation on B independent of α extracts positive energy from B by decreasing the total energy of the two qubits.

The injected energy E_A diffuses to B after a while. The time evolution of the expectation values H_B and V of the average post-measurement state is calculated as

$$\langle H_B(t)\rangle = \sum_\alpha \langle g|P_A(\alpha)|g\rangle\langle A(\alpha)|e^{itH}H_B e^{-itH}|A(\alpha)\rangle$$

$$= \frac{h^2}{2\sqrt{h^2 + k^2}}\left[1 - \cos\left(4kt\right)\right],$$

and

$$\langle V(t)\rangle = \sum_\alpha \langle g|P_A(\alpha)|g\rangle\langle A(\alpha)|e^{itH}V e^{-itH}|A(\alpha)\rangle = 0.$$

Therefore, we enable energy to be extracted from B after a diffusion time scale of $1/k$; this is just the more pedestrian form of energy transportation from A to B. The QET protocol achieves energy transportation from A to B in a time scale much shorter than that of this conventional transportation.

In step II of the protocol, the measurement output α is announced to B. Because the model is nonrelativistic, the propagation speed of the announced output can be much faster than the diffusion speed of the injected energy and can be approximated as infinity. Soon after the arrival of the output α, $U_B(\alpha)$ is performed on B. Then the average state after the operation is given by

$$\rho = \sum_\alpha U_B(\alpha) P_A(\alpha) |g\rangle \langle g| P_A(\alpha) U_B(\alpha)^\dagger.$$

The expectation value of the total energy after the operation is given by

$$\mathrm{Tr}\,[\rho H] = \sum_\alpha \langle g| P_A(\alpha) U_B(\alpha)^\dagger H U_B(\alpha) P_A(\alpha) |g\rangle.$$

On the basis of the fact that $U_B(\alpha)$ commutes with H_A and Eq. (12), the output energy E_B is computed as

$$E_B = E_A - \mathrm{Tr}\,[\rho H] = -\,\mathrm{Tr}\,[\rho\,(H_B + V)]\,.$$

Further, on the basis of the fact that $P_A(\alpha)$ commutes with $U_B(\alpha)$, H_B, and V, the energy can be written as

$$E_B = -\sum_\alpha \langle g| P_A(\alpha)\,(H_B(\alpha) + V(\alpha))\,|g\rangle,$$

where the α-dependent operators are given by

$$H_B(\alpha) = U_B(\alpha)^\dagger H_B U_B(\alpha),$$
$$V(\alpha) = U_B(\alpha)^\dagger V U_B(\alpha).$$

By a straightforward calculation, E_B is computed as

$$E_B = \frac{1}{\sqrt{h^2 + k^2}}\left[hk\sin(2\theta) - \left(h^2 + 2k^2\right)\left[1 - \cos(2\theta)\right]\right]. \qquad (14)$$

Note that $E_B = 0$ if $\theta = 0$, as it should be. If we take a small positive value of θ in Eq. (14), note that E_B takes a small positive value such that

$$E_B \sim \frac{2hk\,|\theta|}{\sqrt{h^2 + k^2}} > 0.$$

Maximization of E_B in terms of θ is achieved by taking a value of θ that satisfies Eqs. (9) and (10). Substituting Eqs. (9) and (10) into Eq. (14) yields the positive value of E_B in Eq. (11). Therefore, even though energy carriers coming from A have not yet arrived at B, the QET protocol can achieve energy extraction from B. As stressed in section 2, the success of energy extraction is due to the emergence of negative energy density at B.

In the section of summary and comment, it will be discussed that a large amount of teleported energy requests a large amount of consumption of the ground-state entanglement between A and B in this model.

Finally a comment is added about extraction and measurement of E_B. As mentioned in the previous section, there is a nontrivial aspect of energy measurements. Some bad measurements suffer from time-energy uncertainty relations and give inevitable error in estimation of E_B. However, we can avoid such a risk by adopting other good measurements of energy. The pointer-basis measurement is one of such good measurements, as stressed in the previous section. Here another setup to measure E_B [5] [7] is reviewed compatible with realistic experiments of QET. The scheme is depicted in figure 23.
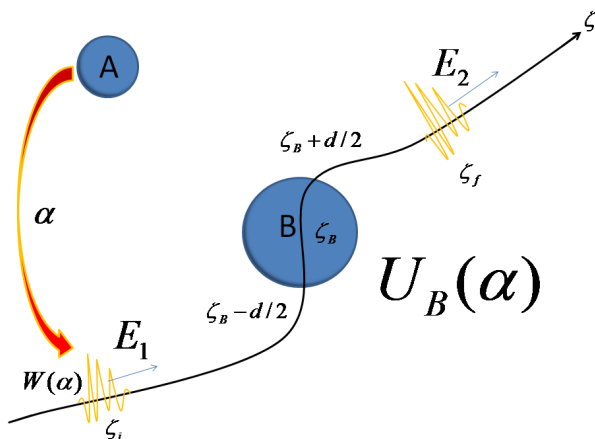

Figure 23: Extraction and measurement of the output energy.

After the arrival of the measurement result α at the region of B, let us generate a laser pulse $W(\alpha)$ in an optical fiber which polarization is dependent on α. A spatial coordinate ζ parametrizes the fiber. The fiber is connected between the generation point of $W(\alpha)$ ($\zeta = \zeta_i$) and the final point ($\zeta = \zeta_f$) via a point $\zeta = \zeta_B$ where spin B stays: $\zeta_i < \zeta_B < \zeta_f$. The pulse $W(\alpha)$ moves toward the final point, and intersects with spin B at $\zeta = \zeta_B$ on the way. Then it can be verified as follows that $W(\alpha)$ performs $U_B(\alpha)$ to B. Let us introduce creation and annihilation bosonic operators $\Psi_\alpha^\dagger(\zeta)$ and $\Psi_\alpha(\zeta)$ for one photon of the laser field with polarization $\alpha = \pm$ in the fiber. The

operators $\Psi_\alpha^\dagger(\zeta)$ and $\Psi_\alpha(\zeta)$ satisfy the following commutation relations:

$$\left[\Psi_\alpha(\zeta),\ \Psi_{\alpha'}^\dagger(\zeta')\right] = \delta_{\alpha\alpha'}\delta\left(\zeta - \zeta'\right),$$
$$\left[\Psi_\alpha(\zeta),\ \Psi_{\alpha'}(\zeta')\right] = 0,$$
$$\left[\Psi_\alpha^\dagger(\zeta),\ \Psi_{\alpha'}^\dagger(\zeta')\right] = 0.$$

The vacuum state $|0\rangle$ of the laser field is defined by

$$\Psi_\alpha(\zeta)|0\rangle = 0.$$

Let us assume that the initial state of the laser field is a pulse-wave coherent state with polarization α given by

$$|\alpha\rangle \propto \exp\left(\int_{-\infty}^{\infty} f_i(\zeta)\Psi_\alpha^\dagger(\zeta)d\zeta\right)|0\rangle,$$

where $f_i(\zeta)$ is the coherent amplitude of the state and a function with a support localized around $\zeta = \zeta_i$. The field strength of the pulse is defined by

$$F = \int_{-\infty}^{\infty} |f_i(\zeta)|^2\, d\zeta.$$

In order to consider a semi-classical coherent state, let us take a large value of F. The free Hamiltonian of the fiber photon reads

$$H_\Psi = -\frac{ic}{2}\int_{-\infty}^{\infty}\left[\Psi(\zeta)^\dagger\partial_\zeta\Psi(\zeta) - \partial_\zeta\Psi(\zeta)^\dagger\Psi(\zeta)\right]d\zeta,$$

where c is the light velocity in the fiber and $\Psi(\zeta)$ is given by

$$\Psi(\zeta) = \left[\begin{array}{c}\Psi_+(\zeta)\\\Psi_-(\zeta)\end{array}\right].$$

The free evolution of the photon field is given by

$$e^{itH_\Psi}\Psi(\zeta)e^{-itH_\Psi} = \Psi(\zeta - ct).$$

The laser field couples with spin B via the interaction given by

$$H_{LO} = \frac{c}{dF}\theta\sigma_B^y\int_{\zeta_B-d/2}^{\zeta_B+d/2}\left[\Psi_+^\dagger(\zeta)\Psi_+(\zeta) - \Psi_-^\dagger(\zeta)\Psi_-(\zeta)\right]d\zeta, \tag{15}$$

where d is the width of the interaction region. The total Hamiltonian of the composite system is expressed as

$$H_{tot} = H + H_{LO} + H_\Psi,$$

and conserved in time. Before the intersection of $W(\alpha)$ with B, the initial state of the composite system is given by

$$\frac{P_A(\alpha)|g\rangle\langle g|P_A(\alpha)}{\langle g|P_A(\alpha)|g\rangle} \otimes |\alpha\rangle\langle\alpha|.$$

In this model, the evolution of the laser pulse induces effective switching of the interaction for $U_B(\alpha)$. In fact, the interaction H_{LO} in Eq. (15) is active only when the pulse exists inside $[\zeta_B - d/2, \zeta_B + d/2]$. Because $W(\alpha)$ is a semi-classical coherent pulse with large F, the photon field can be treated as an external classical field for B in the leading approximation. The switching process for B is described by an effective interaction Hamiltonian as $\langle\alpha|H_{LO}(t)|\alpha\rangle$, where $H_{LO}(t) = e^{itH_\Psi}H_{LO}e^{-itH_\Psi}$. Assuming that the width of the pulse form $f_i(\zeta)$ is much smaller than d, $\langle\alpha|H_{LO}(t)|\alpha\rangle$ can be approximated as

$$\langle\alpha|H_{LO}(t)|\alpha\rangle \sim \frac{c}{d}\Theta\left(\frac{d}{2c} - |t|\right)\theta\sigma_B^y\frac{\alpha}{F}\int_{-\infty}^{\infty}|f_i(\zeta)|^2\,d\zeta \sim \delta(t)\,\alpha\theta\sigma_B^y$$

by taking the nonrelativistic limit $(c \sim \infty)$. Then the time evolution operator of B induced by this effective interaction is calculated as

$$\mathrm{T}\exp\left[-i\int_{-0}^{+0}\langle\alpha|H_{LO}(t)|\alpha\rangle dt\right] = \exp\left[-i\alpha\theta\sigma_B^y\right] = U_B(\alpha).$$

Thus, the interaction in Eq. (15) certainly reproduces the operation $U_B(\alpha)$. The energy of $W(\alpha)$ changes when the pulse passes through $[\zeta_B - d/2, \zeta_B + d/2]$. The average energy of the two-spin system before the interaction with the pulse is E_A. The initial averaged energy of the pulse is denoted by E_1. The average energy of the two-spin system after the interaction becomes $E_A - E_B$ because $U_B(\alpha)$ is operated to B and the energy decreases as the QET effect. The averaged pulse energy after the interaction is denoted by E_2. Then the conservation of H_{tot} ensures that

$$E_A + E_1 = (E_A - E_B) + E_2, \tag{16}$$

because H_{LO} has no contribution in the initial and final state of the scattering process between B and the pulse. Using Eq. (16), the output energy of QET can be rewritten as $E_B = E_2 - E_1$. Consequently, by measuring the initial and final energy of the pulse many times and taking the averages, we can precisely determine the output energy of QET without any problems caused by time-energy uncertainty relations.

4 General Theory of Quantum Energy Teleportation

In this section, the general theory of QET is introduced for one-dimensional discrete chain models. The model is a system composed of many quantum subsystems of general types arrayed in one dimension. The subsystems, labeled by site numbers n, are coupled with each other via nearest-neighbor interactions, as depicted in figure 24.
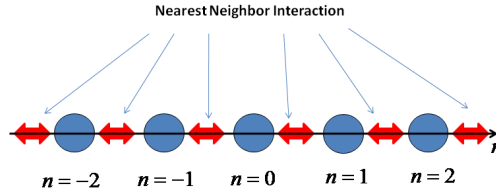

Figure 24: One-dimensional discrete chain model.

We adopt $\hbar = 1$ unit and concentrate on a short time scale during which dynamical evolution induced by the Hamiltonian H is negligible. Let us denote the difference between the largest and smallest eigenvalues of H by ΔE. The timescale t discussed here is assumed to satisfy

$$t \ll \frac{1}{\Delta E}. \tag{17}$$

32

Assuming this condition, it is valid to treat the time evolution operator as $\exp[-itH] \sim I$. It should also be noted that the condition in Eq. (17) can be weakened in the case that a finite amount of energy E_{in} less than ΔE is inputted to the system in the ground state, as follows:

$$t \ll \frac{1}{E_{in}}.$$

In addition, let us assume that LOCC can be repeated for the subsystems many times even in the short time interval. If the site number difference between two parties in the protocol is given by Δn and the chain spacing between nearest-neighbor sites is given by a, the time scale condition for many-round LOCC is expressed as

$$t \gg a\Delta n/c, \tag{18}$$

where c is the velocity of light. By taking the nonrelativistic limit $c \to \infty$, the relation in Eq.(18) always holds.

The energy density operators are Hermitian operators and take the general forms of

$$T_n = X_n + \sum_j \left(\frac{1}{2} g_{n-1/2,j} Y_{n-1,j} Y_{n,j} + \frac{1}{2} g_{n+1/2,j} Y_{n,j} Y_{n+1,j} \right), \tag{19}$$

where X_n and $Y_{n,j}$ are local operators for the subsystem at site n, and $g_{n\pm1/2,j}$ are real coupling constants for the nearest-neighbor interaction. The total Hamiltonian is given by a site-sum of T_n:

$$H = \sum_n T_n.$$

The ground state $|g\rangle$ is the eigenstate of H with the lowest eigenvalue. If the expectation values of T_n do not vanish for the ground state as

$$\langle g|T_n|g\rangle = \epsilon_n \neq 0,$$

we shift the operator X_n on the right-hand side of Eq. (19) by the constant ϵ_n as

$$X'_n = X_n - \epsilon_n,$$

without changing physics. Then, without loss of generality, we can assume the relation given by

$$\langle g|T_n|g\rangle = 0. \tag{20}$$

Eq. (20) derives that the eigenvalue of H for the ground state is zero:

$$H|g\rangle = 0. \tag{21}$$

This is because the eigenvalue is equal to $\langle g|H|g\rangle$ and the following relation holds:

$$\langle g|H|g\rangle = \sum_n \langle g|T_n|g\rangle = 0.$$

From Eq. (21), it is ensured that the Hamiltonian H is a nonnegative operator:

$$H \geq 0. \tag{22}$$

Next, let us define a separable ground state. The separable ground state is the ground state that takes the form of a product of each-site pure states such that

$$|g\rangle = \prod_n |g_n\rangle. \tag{23}$$

Here $|g_n\rangle$ is a pure state for the subsystem at site n. This separable ground state satisfies the factorization property. For instance, a two-point function of T_n and a local operator O_m at site m with $|n - m| > 1$ is equal to the product of the one-point functions for the ground state $|g\rangle$:

$$\langle g|T_n O_m|g\rangle = \langle g|T_n|g\rangle\langle g|O_m|g\rangle. \tag{24}$$

It is well known that the relation in Eq. (24) is often broken for ordinary quantum systems. This implies that the ground states of standard many-body systems are usually nonseparable, and do not satisfy Eq. (23). Such a nonseparable ground state with

$$|g\rangle \neq \prod_n |g_n\rangle. \tag{25}$$

is called an entangled ground state. In the entangled ground state, quantum fluctuations of subsystems share a quantum correlation, that is, entanglement. Though entanglement is an informational concept, it is simultaneously

34

a *physical* resource of quantum communication. For a detailed explanation, see the text book by [21].

If a ground state satisfies the relation of broken factorization,

$$\langle g|T_nO_m|g\rangle \neq \langle g|T_n|g\rangle\langle g|O_m|g\rangle \tag{26}$$

for the energy density at site n and a local operator O_m with $|n - m| > 1$, then the ground state is entangled. This ground-state entanglement leads to an interesting result. It can be proven by use of entanglement that the energy density T_n takes a negative value even though the total Hamiltonian is nonnegative. In order to verify this, let us first prove a useful lemma: The lemma states that if the entangled ground state $|g\rangle$ satisfies the relation in Eq. (26), then $|g\rangle$ is not an eigenstate of T_n. This is because assuming $T_n|g\rangle = \varepsilon_n|g\rangle$ with an eigenvalue ε_n leads to the factorization in Eq. (24) and contradicts Eq. (26). In fact, using $\langle g|T_n = \varepsilon_n\langle g|$ and $\varepsilon_n = \langle g|T_n|g\rangle$, we can directly derive Eq. (24) as follows.

$$\langle g|T_nO_m|g\rangle = \varepsilon_n\langle g|O_m|g\rangle = \langle g|T_n|g\rangle\langle g|O_m|g\rangle.$$

By use of this lemma, we next show that the lowest eigenvalue $\epsilon_-(n)$ of T_n is negative. The operator T_n can be spectrally decomposed into

$$T_n = \sum_{\nu,k_\nu} \epsilon_\nu(n)|\epsilon_\nu(n), k_\nu, n\rangle\langle\epsilon_\nu(n), k_\nu, n|,$$

where $\epsilon_\nu(n)$ are eigenvalues of T_n; $|\epsilon_\nu(n), k_\nu, n\rangle$ are corresponding eigenstates in the total Hilbert space of the chain system; and the index k_ν denotes the degeneracy freedom of the eigenvalue $\epsilon_\nu(n)$. The ground state can be expanded as

$$|g\rangle = \sum_{\nu,k_\nu} g_{\nu,k_\nu}(n)|\epsilon_\nu(n), k_\nu, n\rangle,$$

where $g_{\nu,k_\nu}(n)$ are complex coefficients. Using this expansion, Eq. (20) is rewritten as

$$\sum_{\nu,k_\nu} \epsilon_\nu(n)\left|g_{\nu,k_\nu}(n)\right|^2 = 0. \tag{27}$$

If $\epsilon_-(n)$ is positive, Eq. (27) clearly has no solution for $g_{\nu,k_\nu}(n)$; thus, it is impossible. If $\epsilon_-(n)$ is zero, then Eq. (27) has a solution with nonvanishing $g_{-,k_-}(n)$. Because all the other coefficients $g_{\nu,k_\nu}(n)$ must vanish, this means

that $|g\rangle$ is an eigenstate of T_n with $\epsilon_-(n) = 0$. Therefore, this contradicts Eq. (26) via the above lemma. Therefore, $\epsilon_-(n)$ must be negative:

$$\epsilon_-(n) = -|\epsilon_-(n)| < 0.$$

The average energy density for $|\epsilon_-(n), k_-, n\rangle$ also becomes negative. It is thereby verified that there exist quantum states with negative energy density. It should be stressed that even if a state has negative energy density over a certain region, there exists compensating positive energy density in other regions such that the total energy is not negative, because of the non-negativity of H.

In the later discussion, we adopt general measurements beyond ideal (projective) measurements. Here, let us give a brief overview of the general measurements, which are usually called positive operator valued measure (POVM) measurements. Let us first consider a quantum system S in a state $|\psi\rangle_S$ about which we wish to obtain information. That is, S is the target system of the measurement. In order to execute quantum measurements, we need another quantum system P as a probe. Initially, P is in a state $|0\rangle_P$. In general, the dimensionality of the Hilbert space of S is not equal to that of P. We bring P into contact with S via measurement interactions between the two. In this process, information about $|\psi\rangle_S$ is imprinted into P. After switch-off of the measurement interactions and subsequent signal amplification of the probe system, the total system is in an entangled state that takes a form

$$|\Psi\rangle_{SP} = \sum_{n,\mu} c_{n\mu} |n\rangle_S |\mu\rangle_P.$$

Here, $\{|n\rangle_S\}$ is a complete set of orthonormal basis state vectors of S, and $\{|\mu\rangle_P\}$ is a set of orthonormal state of P. The coefficient $c_{n\mu}$ depends on the initial state $|\psi\rangle_S$ of S. For the state $|\Psi\rangle_{SP}$, a projective measurement detecting which $|\mu\rangle_P$ is realized for P is performed in order to obtain imprinted information about $|\psi\rangle_S$. This completes a general measurement. A conceptual diagram of the measurement is given in figure 25.
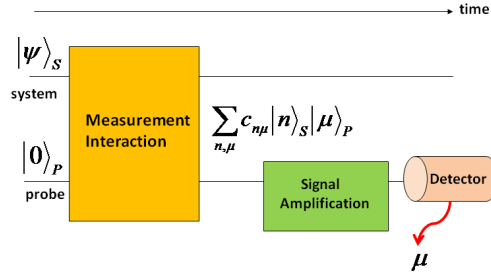
Figure 25: Illustration of POVM measurement.

The emergence probability of μ is given by

$$p_\mu = \sum_n |c_{n\mu}|^2,$$

which is dependent on $|\psi\rangle_S$. Such a general measurement can be always described using measurement operators $M_S(\mu)$ [21], which act on the Hilbert space of S and satisfy

$$\sum_\mu M_S(\mu)^\dagger M_S(\mu) = I_S,$$

where the number of $M_S(\mu)$ is not generally equal to the number of dimensions of the Hilbert space of S. It should be stressed that in general, $M_S(\mu)$ is not a projective Hermitian operator. It can be shown that for an arbitrary quantum state ρ_S of S, the emergence probability $p(\mu)$ of μ can be calculated as

$$p(\mu) = \mathrm{Tr}\left[\rho_S M_S(\mu)^\dagger M_S(\mu)\right].$$

The post-measurement state of S can be computed as

$$\rho(\mu) = \frac{M_S(\mu)\rho_S M_S(\mu)^\dagger}{\mathrm{Tr}\left[\rho_S M_S(\mu)^\dagger M_S(\mu)\right]}.$$

In mathematics, the set of Hermitian positive semidefinite operators $M_S(\mu)^\dagger M_S(\mu)$ is called positive operator valued measure (POVM for short). This is why the general measurement is often called POVM measurement.

Next, let us construct a QET protocol with a discrete chain system. Let us assume that Alice stays in front of one subsystem A at $n = n_A$, and Bob stays in front of another subsystem B at $n = n_B$ with $|n_A - n_B| \geq 5$. Because they are sufficiently separated from each other, it is not only the local operators of A but also the energy density operator T_{n_A}, which is semi-local, that commute with local operators of B and T_{n_B}. At the first step of the QET protocol, Alice performs a POVM measurement on A, which is described by measurement operators $M_A(\alpha)$ with output α satisfying the sum rules given by

$$\sum_\alpha M_A(\alpha)^\dagger M_A(\alpha) = I_A. \tag{28}$$

The POVM of this measurement is defined by

$$\Pi_A(\alpha) = M_A(\alpha)^\dagger M_A(\alpha) \tag{29}$$

Then emergence probability of α is computed for the ground state as

$$p_A(\alpha) = \langle g|\Pi_A(\alpha)|g\rangle.$$

The post-measurement state corresponding to α is given by

$$|A(\alpha)\rangle = \frac{1}{\sqrt{p_A(\alpha)}} M_A(\alpha)|g\rangle.$$

The average post-measurement state is calculated as

$$\rho_M = \sum_\alpha p_A(\alpha)|A(\alpha)\rangle\langle A(\alpha)| = \sum_\alpha M_A(\alpha)|g\rangle\langle g|M_A(\alpha)^\dagger.$$

Therefore, the expectation value of total energy after the measurement is evaluated as

$$E_A = \mathrm{Tr}[H\rho_M] = \sum_\alpha \langle g|M_A(\alpha)^\dagger H M_A(\alpha)|g\rangle.$$

Due to the passivity of $|g\rangle$, E_A is positive. Thus, the measurement device injects energy E_A into the chain system during the measurement. E_A is the input energy of the QET protocol. Because we consider a short time scale for the QET protocol, time evolution of the chain system can be neglected. Hence, the input energy E_A is localized around site n_A after the measurement. To see this directly, let us introduce a local energy operator H_A around site

n_A by the sum of energy density operators that include contributions from A:

$$H_A = \sum_{n=n_A-1}^{n_A+1} T_n.$$

Let us also define an energy operator outside of n_A as

$$H_{\bar{A}} = H - H_A.$$

Then E_A can be computed as

$$E_A = \sum_{\alpha} \langle g | M_A(\alpha)^{\dagger} H_A M_A(\alpha) | g \rangle$$
$$+ \sum_{\alpha} \langle g | M_A(\alpha)^{\dagger} H_{\bar{A}} M_A(\alpha) | g \rangle. \qquad (30)$$

Because of the commutation relation given by

$$[H_{\bar{A}}, \ M_A(\alpha)] = 0,$$

the second term on the right-hand side in Eq. (30) vanishes as follows.

$$\sum_{\alpha} \langle g | M_A(\alpha)^{\dagger} H_{\bar{A}} M_A(\alpha) | g \rangle$$
$$= \langle g | \left(\sum_{\alpha} M_A(\alpha)^{\dagger} M_A(\alpha) \right) H_{\bar{A}} | g \rangle$$
$$= \langle g | H_{\bar{A}} | g \rangle = \sum_{n \notin [n_A-1, n_A+1]} \langle g | T_n | g \rangle$$
$$= 0.$$

Here, we have used Eqs. (28) and (20). Therefore, E_A is equal to the average local energy around site n_A:

$$E_A = \sum_{\alpha} \langle g | M_A(\alpha)^{\dagger} H_A M_A(\alpha) | g \rangle. \qquad (31)$$

It is also verified that the expectation values of T_n with $|n - n_A| \geq 2$ remain exactly zero after the measurement. This ensures that the input energy E_A is stored locally around site n_A. The measurement step is schematically depicted in figure 26.
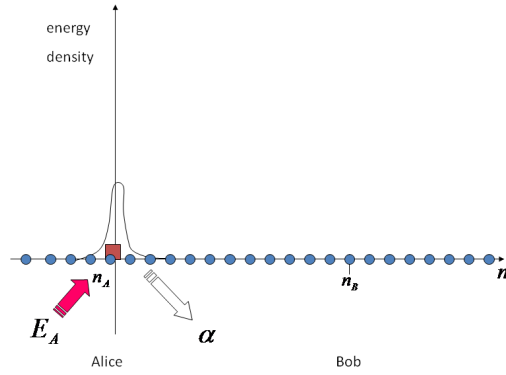
At the second step, Alice announces the measurement result α to Bob via a classical channel. We can assume that the speed of communication is greater than that of energy diffusion of the system. Thus, time evolution of the system is omitted. The second step is schematically depicted in figure 27.
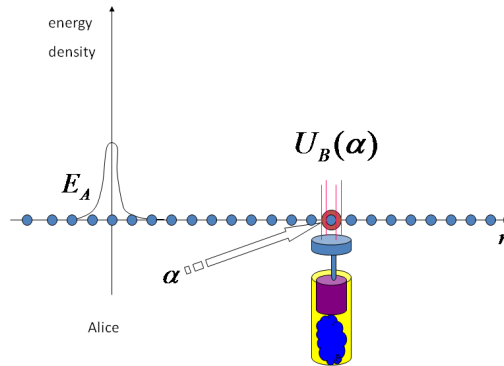
At the third step, Bob performs a local operation $U_B(\alpha)$ dependent on α on B. $U_B(\alpha)$ is given by

$$U_B(\alpha) = \exp\left[-i\alpha\theta G_B\right], \tag{32}$$

40

where G_B is a local Hermitian operator on B and θ is a real constant set such that a positive amount of energy is teleported via QET. After the operation, the average state of the chain system becomes

$$\rho_{QET} = \sum_\alpha U_B(\alpha) M_A(\alpha) |g\rangle \langle g| M_A(\alpha)^\dagger U_B(\alpha)^\dagger. \tag{33}$$

The amount of energy extracted from the chain during the operation is given by

$$E_B = E_A - \mathrm{Tr}\left[H \rho_{QET}\right].$$

This is the output energy of the QET protocol. Later, let us evaluate E_B. Substituting Eq. (33) into the above equation yields

$$E_B = E_A - \sum_\alpha \langle g| M_A(\alpha)^\dagger U_B(\alpha)^\dagger H U_B(\alpha) M_A(\alpha) |g\rangle. \tag{34}$$

As in the case of H_A, let us introduce, for convenience, a local energy operator around site n_B as

$$H_B = \sum_{n=n_B-1}^{n_B+1} T_n.$$

In addition, the energy operator $H_{\overline{AB}}$ outside n_A and n_B is defined as

$$H_{\overline{AB}} = H - H_A - H_B.$$

By this definition, we can derive the following commutation relations because of operator locality.

$$[H_{\overline{AB}}, \ M_A(\alpha)] = 0, \tag{35}$$
$$[H_{\overline{AB}}, \ U_B(\alpha)] = 0. \tag{36}$$

Because the total Hamiltonian H in Eq. (34) is given by a sum of H_A, H_B and $H_{\overline{AB}}$, we obtain the following relation.

$$\begin{aligned}
E_B = E_A &- \sum_\alpha \langle g| M_A(\alpha)^\dagger U_B(\alpha)^\dagger H_A U_B(\alpha) M_A(\alpha) |g\rangle \\
&- \sum_\alpha \langle g| M_A(\alpha)^\dagger U_B(\alpha)^\dagger H_B U_B(\alpha) M_A(\alpha) |g\rangle \\
&- \sum_\alpha \langle g| M_A(\alpha)^\dagger U_B(\alpha)^\dagger H_{\overline{AB}} U_B(\alpha) M_A(\alpha) |g\rangle.
\end{aligned} \tag{37}$$

The second term on the right-hand side of Eq. (37) can be computed as

$$-\sum_\alpha \langle g| M_A\left(\alpha\right)^\dagger U_B(\alpha)^\dagger H_A U_B(\alpha) M_A\left(\alpha\right) |g\rangle$$

$$= -\sum_\alpha \langle g| M_A\left(\alpha\right)^\dagger U_B(\alpha)^\dagger U_B(\alpha) H_A M_A\left(\alpha\right) |g\rangle$$

$$= -\sum_\alpha \langle g| M_A\left(\alpha\right)^\dagger H_A M_A\left(\alpha\right) |g\rangle = -E_A,$$

where we have used an operator locality relation given by $[H_A,\ U_B(\alpha)] = 0$, the unitary relation of $U_B(\alpha)$, and Eq. (31). This term compensates for the first term (E_A) and has no contribution. The fourth term on the right-hand side of Eq. (37) vanishes as follows.

$$\sum_\alpha \langle g| M_A\left(\alpha\right)^\dagger U_B(\alpha)^\dagger H_{\overline{AB}} U_B(\alpha) M_A\left(\alpha\right) |g\rangle$$

$$= \sum_\alpha \langle g| M_A\left(\alpha\right)^\dagger U_B(\alpha)^\dagger U_B(\alpha) H_{\overline{AB}} M_A\left(\alpha\right) |g\rangle$$

$$= \sum_\alpha \langle g| M_A\left(\alpha\right)^\dagger H_{\overline{AB}} M_A\left(\alpha\right) |g\rangle$$

$$= \langle g| \left(\sum_\alpha M_A\left(\alpha\right)^\dagger M_A\left(\alpha\right) \right) H_{\overline{AB}} |g\rangle$$

$$= \langle g| H_{\overline{AB}} |g\rangle = \langle g| H |g\rangle - \sum_{n=n_A-1}^{n_A+1} \langle g| T_n |g\rangle - \sum_{n=n_B-1}^{n_B+1} \langle g| T_n |g\rangle$$

$$= 0$$

Here, we have used Eq. (36), the unitary relation of $U_B(\alpha)$, Eq. (35), Eq. (28), and Eq. (20) in this order. Therefore, E_B is equal to the third term on the right-hand side of Eq. (37):

$$E_B = -\sum_\alpha \langle g| M_A\left(\alpha\right)^\dagger U_B(\alpha)^\dagger H_B U_B(\alpha) M_A\left(\alpha\right) |g\rangle.$$

Because $U_B(\alpha)^\dagger H_B U_B(\alpha)$ commutes with $M_A\left(\alpha\right)$ due to the operator locality, we can rewrite E_B as

$$E_B = -\sum_\alpha \langle g| \Pi_A\left(\alpha\right) U_B(\alpha)^\dagger H_B U_B(\alpha) |g\rangle \tag{38}$$

42

by using Eq. (29). The output energy E_B in Eq. (38) can always take a positive value by properly selecting the value of θ in Eq. (32). In fact, let us consider a case with a small value of $|\theta|$ in which $U_B(\alpha)$ can be approximated as

$$U_B(\alpha) = \exp\left[-i\theta\alpha G_B\right] \sim 1 - i\theta\alpha G_B.$$

Then E_B is evaluated as

$$E_B = -\langle g|\left(\sum_\alpha M_A(\alpha)^\dagger M_A(\alpha)\right) H_B|g\rangle$$
$$+ i\theta\langle g|\left(\sum_\alpha \alpha\Pi_A(\alpha)\right)[H_B,\ G_B]|g\rangle + O(\theta^2).$$

The first term in the above equation vanishes because of Eq. (28) and Eq. (20). Therefore, E_B is written as

$$E_B = \theta\langle g|D_A\dot{G}_B|g\rangle + O(\theta^2),$$

where D_A is a Hermitian operator given by

$$D_A = \sum_\alpha \alpha\Pi_A(\alpha)$$

and $\dot{G}_B$ is a Hermitian operator given by

$$\dot{G}_B = i[H_B,\ G_B] = i[H,\ G_B].$$

By this definition, $\dot{G}_B$ denotes the time derivative operator of the Heisenberg operator $G_B(t) = e^{itH}G_B e^{-itH}$ at $t = 0$. Let us introduce a significant parameter η as a two-point function for the ground state, given by

$$\eta = \langle g|D_A\dot{G}_B|g\rangle.$$

The reality of η ($\eta^* = \eta$) is guaranteed by an operator locality relation given by $\left[D_A,\ \dot{G}_B\right] = 0$. Then E_B is simply written as

$$E_B = \theta\eta + O(\theta^2).$$

43

As long as η does not vanish, E_B can take a positive value;

$$E_B \sim |\theta\eta| > 0$$

by taking the same sign of θ as that of η:

$$\theta = \mathrm{sgn}\,(\eta)\,|\theta|\,.$$

Therefore, it is really possible to teleport a positive amount of energy by this protocol. It is also verified that local energy around site n_B takes a negative value $-E_B$ because of local energy conservation. The last step of the protocol is depicted schematically in figure 28.
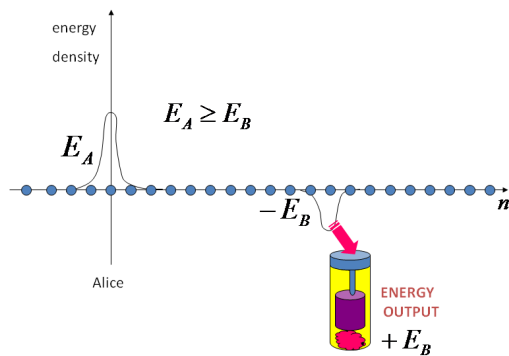
Maximization of E_B in terms of θ should be independently performed for each QET model. For example, in a protocol with a qubit chain, we have the following result [4]-[5]. The qubit chain is composed of qubits arrayed in one dimension and coupled by nearest-neighbor interactions. In the model, Alice measures a local observable σ_A given by a component of the Pauli operator in the direction of a three-dimensional unit real vector $\vec{u}_A$ such that

$$\sigma_A = \vec{u}_A \cdot \vec{\sigma}_{n_A}.$$

The outputs α are its eigenvalue ± 1, and projective operators onto the eigenstates corresponding to α are denoted by $P_A(\alpha)$. Then the input energy is given by

44

$$E_A = \sum_\alpha \langle g | P_A(\alpha) H P_A(\alpha) | g \rangle.$$

The local generator G_B of Bob's operation is given by a component σ_B of the Pauli operator in the direction of a three-dimensional unit real vector $\vec{u}_B$ such that

$$G_B = \sigma_B = \vec{u}_B \cdot \vec{\sigma}_{n_B}.$$

Because the eigenvalues of G_B are ± 1, the square of G_B becomes the identical operation of B:

$$G_B^2 = I_B.$$

Therefore, we can calculate $U_B(\alpha)$ without approximation as

$$\begin{aligned} U_B(\alpha) &= \exp\left[-i\theta\alpha G_B\right] \\ &= I_B \cos\theta - i\alpha\sigma_B \sin\theta. \end{aligned} \tag{39}$$

Using Eq. (39), E_B in Eq. (38) is explicitly computed as

$$E_B = \frac{\eta}{2}\sin(2\theta) - \frac{\xi}{2}\left(1 - \cos(2\theta)\right), \tag{40}$$

where η is given by

$$\eta = \langle g | \sigma_A \dot{\sigma}_B | g \rangle$$

with $D_A = \sigma_A$ and $G_B = \sigma_B$. The constant ξ is defined as

$$\xi = \langle g | \sigma_B H \sigma_B | g \rangle,$$

and is positive. Maximization of E_B in terms of θ is achieved by fixing θ as

$$\cos(2\theta) = \frac{\xi}{\sqrt{\xi^2 + \eta^2}}, \tag{41}$$

$$\sin(2\theta) = \frac{\eta}{\sqrt{\xi^2 + \eta^2}}. \tag{42}$$

Substituting Eqs. (41) and (42) into Eq. (40) yields the maximum value of E_B such that

$$E_B = \frac{1}{2}\left[\sqrt{\xi^2 + \eta^2} - \xi\right].$$

As long as η is nonzero, E_B becomes positive.

As a significant qubit chain model, let us consider the critical Ising model with transverse magnetic field. It has energy density at site n

$$T_n = -J\sigma_n^z - \frac{J}{2}\sigma_n^x \left(\sigma_{n+1}^x + \sigma_{n-1}^x\right) - \epsilon,$$

where J is a positive constant and ϵ is a real constant satisfying $\langle g|T_n|g\rangle = 0$. The total Hamiltonian reads

$$H = \sum_n T_n = -J\left[\sum_{n=-\infty}^{\infty} \sigma_n^z + \sum_{n=-\infty}^{\infty} \sigma_n^x\sigma_{n+1}^x\right] - E_g, \tag{43}$$

where E_g is a constant that shifts the eigenvalue of the ground state $|g\rangle$ to zero. Let us take $\vec{u}_A = (0,1,0)^T$ and $\vec{u}_B = (1,0,0)^T$. By the standard treatment of the model, we can analytically evaluate Alice's input energy [4] as

$$E_A = \frac{6}{\pi}J.$$

Meanwhile, Bob's output energy is evaluated [4] as

$$E_B = \frac{2J}{\pi}\left[\sqrt{1 + \left(\frac{\pi}{2}\Delta(|n_A - n_B|)\right)^2} - 1\right],$$

where the function $\Delta(n)$ is defined by

$$\Delta(n) = -\left(\frac{2}{\pi}\right)^n \frac{2^{2n(n-1)}h(n)^4}{(4n^2 - 1)\,h(2n)}$$

with

$$h(n) = \prod_{k=1}^{n-1} k^{n-k}.$$

When we take a large separation between Alice and Bob ($|n_B - n_A| \gg 1$), it is straightforwardly verified that the decay of E_B obeys not an exponential but a power law because of the criticality of this model. In fact, E_B takes an asymptotic form of

$$E_B \sim J\frac{\pi}{64}\sqrt{e}2^{1/6}c^{-6}\,|n_B - n_A|^{-9/2}, \tag{44}$$

where the constant c is evaluated as $c \sim 1.28$.

46

Note that the input energy E_A is still stored around Alice even after the last step of the QET protocol. What happens if Alice attempts to completely withdraw E_A by local operations at site n_A after the energy extraction by Bob? If this was possible, the energy gain E_B of Bob might have no cost. However, if so, the total energy of the chain system becomes equal to $-E_B$ and negative, as depicted in figure 29.
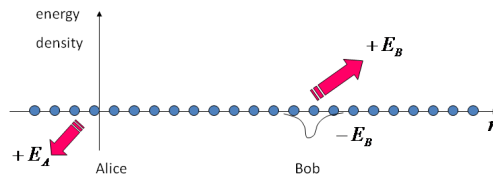

Figure 29: Schematic figure showing impossibility of withdrawing the input energy completely by Alice.

Meanwhile, we know that the total energy must be nonnegative. Hence, Alice cannot withdraw energy larger than $E_A - E_B$ only by her local operations. The main reason for Alice's failure is that the first local measurement of A breaks the ground-state entanglement between A and all the other subsystems. In particular, in the case with projective measurements, the post-measurement state is an exact separable state with no entanglement between A and the other subsystems. If Alice wants to recover the original state with zero energy, she must recreate the broken entanglement. However, entanglement generation, in general, needs nonlocal operations [21]. Therefore, Alice cannot recover the state perfectly by her local operations alone. Thus, a residual energy inevitably remains around A inside the chain system. This interesting aspect poses a related problem about residual energy of local cooling. Let us imagine that we stop the QET protocol soon after Alice's measurement, and attempt to completely withdraw E_A by local operations. By the same argument as above, it can be shown that this attempt never succeeds because the measurement already breaks the ground-state entan-

47

glement. As in the upper part of figure 30, A at site n_A is entangled with other subsystems in the ground state with zero energy.
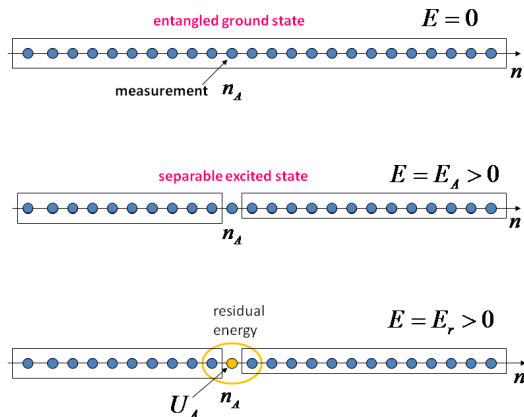

Figure 30: In the upper part of the figure, a subsystem A is entangled between other subsystems in the ground state with zero energy. This entanglement is broken by the measurement of A and A jumps into a pure state. The post-measurement state of the chain system is not the ground state but instead an excited state carrying positive energy as depicted in the middle part of the figure. Even if an arbitrary local operation is performed on A, the broken entanglement cannot be recovered and nonvanishing energy remains inside the chain system, as depicted in the lower part of the figure.

This entanglement is broken by the projective measurement of A, when A jumps into a pure state. The post-measurement state of the chain system is not the ground state but instead an excited state carrying E_A, as depicted in the middle part of figure 30. Even if an arbitrary local operation U_A is performed on A, the broken entanglement cannot be recovered and nonvanishing energy remains inside the chain system, as depicted in the lower part of figure 30. For a long time interval beyond the short-time scale of this protocol, it is actually possible to extract E_A by local operations with the assistance of dynamical evolution induced by the nonlocal Hamiltonian H. However, in the short time interval we considered, this dynamical evolution is not available. Therefore, we conclude that the minimum value E_r with respect to short-time local-cooling operations is always positive. In order to make the argument more concrete, let us consider a general local-cooling operation on A after Alice's measurement obtaining the measurement result α. It is known [21] that the operation is generally expressed by the use of

α-dependent local Kraus operators $K_A(\alpha, \mu)$ at site n_A satisfying

$$\sum_\mu K_A^\dagger(\alpha, \mu) K_A(\alpha, \mu) = I_A. \tag{45}$$

Then the quantum state after this local cooling of A is given by

$$\rho_c = \sum_{\mu,\alpha} K_A(\alpha, \mu) M_A(\mu) |g\rangle\langle g| M_A^\dagger(\mu) K_A^\dagger(\alpha, \mu). \tag{46}$$

The minimum value E_r of the residual energy in terms of $K_A(\alpha, \mu)$ satisfying Eq. (45) is defined as

$$E_r = \min_{\{K_A(\alpha,\mu)\}} \text{Tr}\left[\rho_c H\right]. \tag{47}$$

For example, evaluation of E_r is performed analytically in the critical Ising spin model in Eq. (43). The result is obtained in [4] and given by

$$E_r = \left(\frac{6}{\pi} - 1\right) J > 0.$$

Alice cannot extract this energy by any short-time local operation, even though it really exists in front of her. Because of the nonnegativity of H, it is easily noticed that E_r is lower bounded by the teleported energy E_B.

It is worth noting that energy can be extracted simultaneously via QET from not only B but also other subsystems if we know the measurement result of A. In figure 31, Alice stays at $n = 0$ and performs the measurement of A.
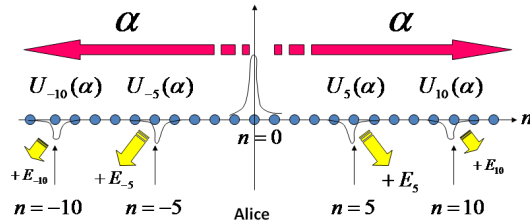

Figure 31: Illustration of quantum energy distribution.

She announces the measurement result α to other sites. Then we can simultaneously extract energy from many distant sites by local unitary operations $U_n(\alpha)$ dependent on α at site n with $|n| \geq 5$. This extended protocol is called quantum energy distribution (QED for short). It is also proven [4] that the energy input E_A during the measurement of A is lower bounded by the sum of the teleported energies extracted from other distant sites. Therefore, analogically speaking from the operational viewpoint, the input energy E_A is stored in a form that can be compared to a broad oil field. If we are authorized users who know the 'password' α, we are able to simultaneously extract energy as *oil* from the quantum system, the *oil field*, at many sites distant from A.

In the conventional forms of energy transportation, impurities in the channel generate heat when the energy carriers pass through the channel. Thus, time scale of energy transportation becomes the same order of that of heat generation. Meanwhile, in QET, because it is not energy but classical information that is sent, the intermediate subsystems along the channel between the sender (Alice) and the receiver (Bob) are not excited by the energy carriers of the system during the short time of a QET process, as depicted in figure 32.
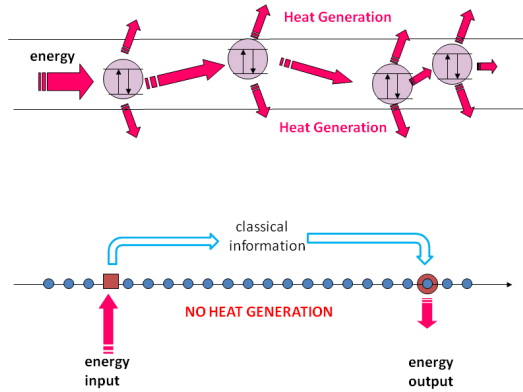

Figure 32: Comparison between more conventional energy transportation and QET.

Much after the transportation, dynamical evolution of the system begins and then heat is generated. Thus, the time scale for effective energy transportation by QET is much shorter than that of heat generation. This property is one of the remarkable advantages of QET. Due to this property, QET is

50

expected to find use as an energy distribution scheme inside quantum devices that avoids thermal decoherence and would thus assist in the development of quantum computers.

Because QET is based on the physics of zero-point fluctuations, which are usually quite small, the amount of teleported energy is generally small. However, in a practical application of QET for nanodevices, it would be possible to consider N discrete chains with one-bit measurements as a single QET channel with N-bit information transfer. Then, the amount of teleported energy is enhanced by the factor N, as depicted in figure 33.
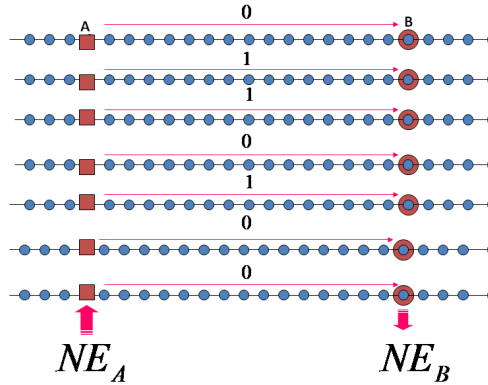

Figure 33: Many discrete chains as a single QET channel.

It is trivial that a large amount of energy can be transported via QET by the use of parallel arrays of many quantum chains.

5 QET with Quantum Field

In section 4, we discussed QET protocols with quantum chains that consist of subsystems discretely arrayed in one dimension. In this section, we treat a QET protocol with a massless relativistic field f in one dimension as a continuum. For a detailed analysis, see [9] and [11]. We adopt the natural unit $c = \hbar = 1$. The equation of motion reads

$$\left[\partial_t^2 - \partial_x^2\right] f = 0. \tag{48}$$

This equation can be exactly solved using the light-cone coordinates such that

$$x^{\pm} = t \pm x.$$

Then Eq. (48) is transformed into

$$\partial_+ \partial_- f = 0.$$

and general solutions of this equation are given by the sum of the left-mover component $f_+ \left(x^+\right)$ and right-mover component $f_- \left(x^-\right)$:

$$f = f_+ \left(x^+\right) + f_- \left(x^-\right).$$

The canonical conjugate momentum operator of $f\left(x\right) = f|_{t=0}$ is defined by

$$\Pi(x) = \partial_t f|_{t=0}$$

and it satisfies the standard commutation relation,

$$\left[f\left(x\right),\ \Pi\left(x'\right)\right] = i\delta\left(x - x'\right).$$

The left-moving wave $f_+ \left(x^+\right)$ can be expanded in terms of plane-wave modes as

$$f_+ \left(x^+\right) = \int_0^\infty \frac{d\omega}{\sqrt{4\pi\omega}} \left[a_\omega^L e^{-i\omega x^+} + a_\omega^{L\dagger} e^{i\omega x^+}\right],$$

where a_ω^L ($a_\omega^{L\dagger}$) is an annihilation (creation) operator of a left-moving particle and satisfies

$$\left[a_\omega^L,\ a_{\omega'}^{L\dagger}\right] = \delta\left(\omega - \omega'\right). \tag{49}$$

The right-moving wave $f_- \left(x^-\right)$ can also be expanded in the same way using the plane-wave modes. The energy density operator is given by

$$\varepsilon(x) = \frac{1}{2} : \Pi(x)^2 : + \frac{1}{2} : \left(\partial_x f(x)\right)^2 :,$$

where :: denotes the normal order of creation–annihilation operators for the plain-wave modes. The Hamiltonian is given by $H = \int_{-\infty}^{\infty} \varepsilon(x) dx$. The

eigenvalue of the vacuum state has been automatically tuned to be zero due to the normal ordering in $\varepsilon(x)$:

$$H|0\rangle = 0.$$

The vacuum state also satisfies

$$a_\omega^L|0\rangle = 0,$$
$$\langle 0|\varepsilon(x)|0\rangle = 0.$$

Let us introduce the chiral momentum operators as

$$\Pi_\pm(x) = \Pi(x) \pm \partial_x f(x).$$

Then the energy density can be rewritten as

$$\varepsilon(x) = \frac{1}{4} : \Pi_+(x)^2 : + \frac{1}{4} : \Pi_-(x)^2 : . \tag{50}$$

We perform a QET protocol for the vacuum state $|0\rangle$ as follows: Let us consider a probe system P of a qubit located in a small compact region $[x_{A-}, x_{A+}]$ satisfying $x_{A-} > 0$ in order to detect zero-point fluctuations of f. In a manner similar to that of Unruh [23], we introduce a measurement Hamiltonian between f and the qubit such that

$$H_m(t) = g(t)G_A \otimes \sigma_y,$$

where $g(t)$ is a time-dependent real coupling constant, G_A is given by

$$G_A = \frac{\pi}{4} + \int_{-\infty}^{\infty} \lambda_A(x)\Pi_+(x)\,dx, \tag{51}$$

$\lambda_A(x)$ is a real function with support $[x_{A-}, x_{A+}]$, and σ_y is the y-component of the Pauli operator of the qubit. Alice stays in the region $[x_{A-}, x_{A+}]$. We assume that the initial state of the qubit is the up state $|+\rangle$ of the z-component σ_z. In the later analysis, we choose a sudden switching form such that $g(t) = \delta(t)$. After the interaction is switched off, we measure the z-component σ_z for the probe spin. If the up or down state, $|+\rangle$ or $|-\rangle$, of σ_z is observed, we assign $\alpha = +$ or $\alpha = -$, respectively, to the measurement result. The measurement is completed at $t = +0$. The time evolution of this

measurement process with output α can be described by the measurement operators $M_A(\alpha)$, which satisfy

$$M_A(\alpha)\rho M_A(\alpha)^\dagger = \operatorname*{Tr}_P \left[(I \otimes |\alpha\rangle\langle\alpha|)\, U(+0)\, (\rho \otimes |+\rangle\langle+|)\, U(+0)^\dagger \right],$$

where ρ is an arbitrary density operator of the field, the time evolution operator $U(+0) = \mathrm{T} \exp\left[-i \int_{-0}^{+0} H_m(t') dt' \right]$ generated by the instantaneous interaction is computed as $\exp\left[-iG_A \otimes \sigma_y \right]$, and the trace Tr_P is taken to the probe system. The measurement operators $M_A(\alpha)$ are evaluated as

$$M_A(\alpha) = \langle\alpha| \exp\left[-iG_A \otimes \sigma_y \right] |+\rangle.$$

Hence, we obtain the explicit expression of $M_A(\alpha)$ such that

$$M_A(+) = \cos G_A, \tag{52}$$
$$M_A(-) = \sin G_A. \tag{53}$$

For the vacuum state $|0\rangle$, the emergence probability of α is independent of α and is given by $1/2$ [11]. The post-measurement states of f for the result α are calculated as

$$|\psi(\alpha)\rangle = \sqrt{2} M_A(\alpha)|0\rangle = \frac{1}{\sqrt{2}} \left(e^{-\frac{\pi}{4}i}|\lambda\rangle + \alpha e^{\frac{\pi}{4}i}|-\lambda\rangle \right), \tag{54}$$

where $|\pm\lambda\rangle$ are left-moving coherent states defined by

$$|\pm\lambda\rangle = \exp\left[\pm i \int_{-\infty}^{\infty} \lambda_A(x)\Pi_+(x)\, dx \right] |0\rangle. \tag{55}$$

The two states $|\psi(+)\rangle$ and $|\psi(-)\rangle$ are nonorthogonal to each other with $\langle\psi(+)|\psi(-)\rangle = \langle\lambda|-\lambda\rangle \neq 0$ because this POVM measurement is not projective [21]. The measurement is depicted schematically in figure 34.
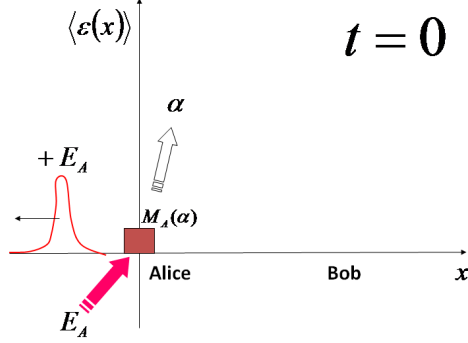
54

The expectation value of the Heisenberg operator of energy density $\varepsilon(x,t)$ for each post-measurement state is independent of α and given by

$$\langle\psi(\alpha)|\varepsilon(x,t)|\psi(\alpha)\rangle = \left(\partial_+\lambda_A(x^+)\right)^2. \tag{56}$$

Hence, the amount of total average excitation energy is time-independent and evaluated as

$$E_A = \int_{-\infty}^{\infty}(\partial_x\lambda_A(x))^2 dx. \tag{57}$$

The average state at time T is expressed as

$$\rho_M = \sum_\alpha e^{-iTH}M_A(\alpha)|0\rangle\langle0|M_A(\alpha)^\dagger e^{iTH}.$$

It is worth noting that the state ρ_M is a strictly localized state defined by Knight [24], because ρ_M is locally the same as $|0\rangle\langle0|$ at $t = T$ and satisfies $\mathrm{Tr}\left[\rho_M\varepsilon(x)\right] = 0$ for $x \notin [x_{A-} - T, x_{A+} - T]$.

Bob stays in the region $[x_{B-}, x_{B+}]$ with zero energy density and is on Alice's right-hand side:

$$x_{A+} < x_{B-}.$$

Alice sends information about the result α, to Bob at $t = +0$ at the speed of light. Bob receives it at $t = T$. This is depicted schematically in figure 35.
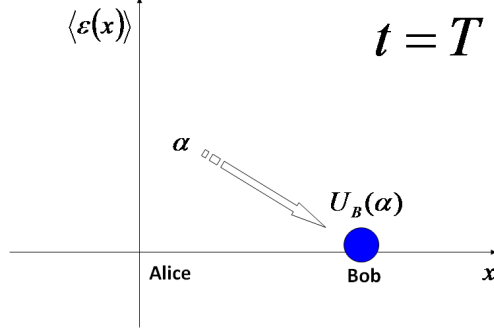
It should be stressed that the positive-energy wave packet generated by the measurement propagates to the left from Alice and the information about α propagates to the right from Alice. Therefore, only classical information is sent from Alice to Bob. The average energy density of quantum fluctuation around Bob remains zero at $t = T$. Then Bob performs a unitary operation on the quantum field f; the unitary operation is dependent on α and is given by

$$U_B(\alpha) = \exp\left[i\alpha\theta \int_{-\infty}^{\infty} p_B(x)\Pi_+\left(x\right)dx\right], \tag{58}$$

where θ is a real parameter fixed below and $p_B(x)$ is a real function of x with its support $[x_{B-}, x_{B+}]$. After the operation, the average state of the field f is given by

$$\rho_F = \sum_\alpha U_B(\alpha)e^{-iTH}M_A(\alpha)|0\rangle\langle0|M_A(\alpha)^\dagger e^{iTH}U_B(\alpha)^\dagger.$$

Let us introduce an energy operator localized around the region $[x_{B-}, x_{B+}]$ such that $H_B = \int_{-\infty}^{\infty} w_B\left(x\right)\varepsilon\left(x\right)dx$. Here, $w_B\left(x\right)$ is a real window function with $w_B(x) = 1$ for $x \in [x_{B-}, x_{B+}]$ and it rapidly decreases outside the region. The average amount of energy around the region is evaluated [11] as

$$\mathrm{Tr}\left[\rho_F H_B\right] = -\theta\eta + \theta^2\xi$$

where $\xi = \int_{-\infty}^{\infty} \left(\partial_x p_B(x)\right)^2 dx$ and

$$\eta = -\frac{4}{\pi} |\langle 0|2\lambda\rangle| \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} p_B(x) \frac{1}{(x-y+T)^3} \lambda_A(y) dx dy. \qquad (59)$$

By fixing the parameter θ such that

$$\theta = \frac{\eta}{2\xi}$$

so as to minimize $\mathrm{Tr}\,[\rho_F H_B]$, it is proven that the average energy around Bob takes a negative value, that is,

$$\mathrm{Tr}\,[\rho_F H_B] = -\frac{\eta^2}{4\xi} < 0. \qquad (60)$$

During the operation by Bob, the total average energy decreases by

$$E_B = \mathrm{Tr}\,[\rho_M H] - \mathrm{Tr}\,[\rho_F H] = E_A - \mathrm{Tr}\,[\rho_F H]. \qquad (61)$$

Because average energy density at $t = T$ vanishes except in the region of the wave packet excited by Alice's measurement and the region of Bob, the following relation is proven straightforwardly.

$$\mathrm{Tr}\,[\rho_F H] = \mathrm{Tr}\,[\rho_F H_A(T)] + \mathrm{Tr}\,[\rho_F H_B], \qquad (62)$$

where $H_A(T) = \int_{-\infty}^{\infty} w_A(x+T)\,\varepsilon(x)\,dx$ and $w_A(x)$ is a real window function for Alice with $w_A(x) = 1$ for $x \in [x_{A-}, x_{A+}]$ and it rapidly decreases outside the region. The term $\mathrm{Tr}\,[\rho_F H_A(T)]$ in Eq. (62) is the contribution of the left-moving positive-energy wave packet generated by Alice's measurement. By virtue of operation locality, it can be proven that the average energy of the wave packet remains unchanged after Bob's operation:

$$\mathrm{Tr}\,[\rho_F H_A(T)] = E_A. \qquad (63)$$

Substituting Eq. (62) with Eq. (63) into Eq. (61) yields

$$E_B = -\,\mathrm{Tr}\,[\rho_F H_B].$$

According to local energy conservation, the same amount of energy is moved from the field fluctuation to external systems, including the device executing

$U_B(\alpha)$. Therefore, E_B is the output energy of this QET protocol. By using the result in Eq. (60), E_B can be evaluated as

$$E_B = \frac{4\,|\langle 0|2\lambda\rangle|^2}{\pi^2} \frac{\left[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} p_B(x)\frac{1}{(x-y+T)^3}\lambda_A(y)dxdy\right]^2}{\int_{-\infty}^{\infty}\left(\partial p_B(x')\right)^2 dx'}.$$

The operation by Bob simultaneously generates a wave packet with negative energy $-E_B$ that propagates toward the left-side spatial infinity. The situation after the final step of the protocol is depicted schematically in figure 36.
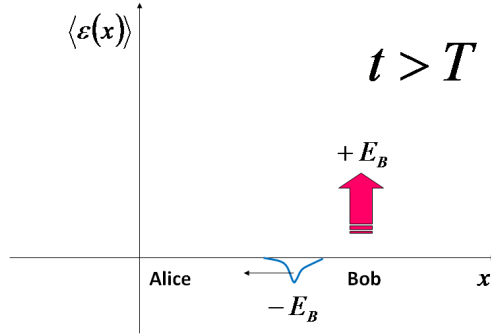

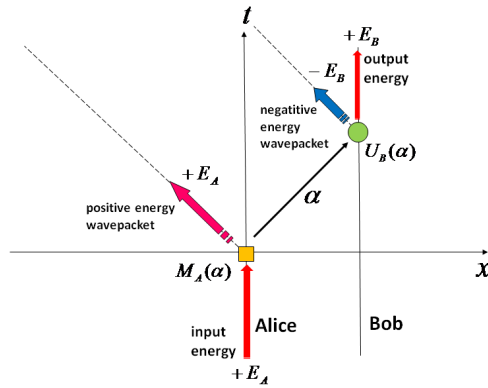Figure 36: Third step of a QET protocol with a quantum field.

Figure 37: Spacetime diagram of a QET protocol with a quantum field.

58

The protocol is summarized in a spacetime diagram in figure 37. Using both this protocol and a previous quantum teleportation protocol in [2], it is possible, in principle, to teleport an object with energy to a zero-energy local-vacuum region like the above region $[x_{B-}, x_{B+}]$.

It is worthwhile to comment that an analogous QET protocol to this protocol would be experimentally implimented by using quantum Hall edge currents [12]. The edge current can be described by a one-dimensional quantum scalar field like the system discussed in this section. The most striking feature of this experimental proposal is that the output energy of the QET protocol may be of order of $100\mu eV$, which can be observed using current technology.

6   Summary and Comment

New protocols for quantum energy teleportation (QET) are reviewed that can accomplish energy transportation by local operations and classical communication. The protocols do not violate any physical laws, including causality and local energy conservation. The salient features of QET are ground-state entanglement of many-body systems and the emergence of negative energy density due to this entanglement. Research on QET is expected to assist in the development of quantum nanodevices, including quantum computers. In addition, QET may shed light on fundamental physics, including quantum Maxwell's demons, phase transition at zero temperature, and the origin of black hole entropy.

Finally, a comment is added on energy–entanglement relations in QET. As a quantitative entanglement measure, negativity is computed between separated blocks of qubit chains [25] (the logarithmic negativity for harmonic oscillator chains [26] [8]) showing that at criticality, this negativity is a function of the ratio of the separation to the length of the blocks and can be written as a product of a power law and an exponential decay. This suggests, for the arguments in section 4, that change in the entanglement between A and B after a local measurement of A has a similar rapid-decay dependence on the spatial separation. Thus, it may be concluded that bipartite entanglement between A and B itself is not essential for QET. Though the bipartite entanglement between the two may be rapidly damped, E_B

shows a power law decay ($\propto n^{-9/2}$) for large spatial separation n at criticality, as seen in Eq. (44). In a sense, this implies that an almost classical correlation between A and B is sufficient to execute QET for large separation, and is expected to be robust against environmental disturbances in contrast to the entanglement fragility in the previous quantum teleportation scheme. It should be emphasized, however, that this classical correlation is *originally induced by the ground-state multipartite entanglement* generated by nearest-neighbor interactions. If the ground state is separable, we have no correlation between A and B. This suggests that teleporting a positive amount of energy requires some amount of ground-state entanglement. In fact, for the minimal model of QET discussed in section 3, we have nontrivial energy–entanglement relations. Let us consider a set of POVM measurements for A which measurement operators $M_A(\mu)$ with measurement output μ commute with the interaction Hamiltonian V in the minimal model. These measurements do not disturb the energy density at B in the ground state. Entropy of entanglement is adopted as a quantitative measure of entanglement. Before the measurement of A, the total system is prepared to be in the ground state $|g\rangle$. The reduced state of B is given by $\rho_B = \mathrm{Tr}_A\left[|g\rangle\langle g|\right]$. The emergent probability $p_A(\mu)$ of μ is given by $\langle g|M_A(\mu)^\dagger M_A(\mu)|g\rangle$. After the POVM measurement outputting μ, the reduced post-measurement state of B is calculated as $\rho_B(\mu) = \frac{1}{p_A(\mu)}\mathrm{Tr}_A\left[M_A(\mu)|g\rangle\langle g|M_A(\mu)^\dagger\right]$. The entropy of entanglement of the ground state is given by $-\mathrm{Tr}_B\left[\rho_B \ln \rho_B\right]$ and that of the post-measurement state with output μ is given by $-\mathrm{Tr}_B\left[\rho_B(\mu)\ln \rho_B(\mu)\right]$. By using these results, we define the consumption of ground-state entanglement by the measurement as the difference between the ground-state entanglement and the averaged post-measurement-state entanglement:

$$\Delta S_{AB} = -\mathop{\mathrm{Tr}}_B\left[\rho_B \ln \rho_B\right] - \sum_\mu p_A(\mu)\left(-\mathop{\mathrm{Tr}}_B\left[\rho_B(\mu)\ln \rho_B(\mu)\right]\right).$$

For any measurement which satisfies $[M_A(\mu),\ V] = 0$, the following relation holds [6]:

$$\Delta S_{AB} \geq \frac{1+\sin^2\varsigma}{2\cos^3\varsigma}\ln\frac{1+\cos\varsigma}{1-\cos\varsigma}\frac{\max E_B}{\sqrt{h^2+k^2}}, \qquad (64)$$

where ς is a real constant fixed by the coupling constants of the minimal

model such that

$$\cos\varsigma = \frac{h}{\sqrt{h^2 + k^2}}, \ \ \sin\varsigma = \frac{k}{\sqrt{h^2 + k^2}}.$$

$\max E_B$ is the maximum output energy of QET in terms of the local operation of B dependent on μ. Eq. (64) implies that a large amount of teleported energy really requests a large amount of consumption of the ground-state entanglement between A and B. It is also noted that for a QET model with a linear harmonic chain, we have a similar relation between teleported energy and entanglement [8]. Consequently, it can be said that the ground-state entanglement really gives birth to QET from the point of view of information theory [27]. The ground-state entanglement is a *physical* resource for energy teleportation.

Acknowledgments

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